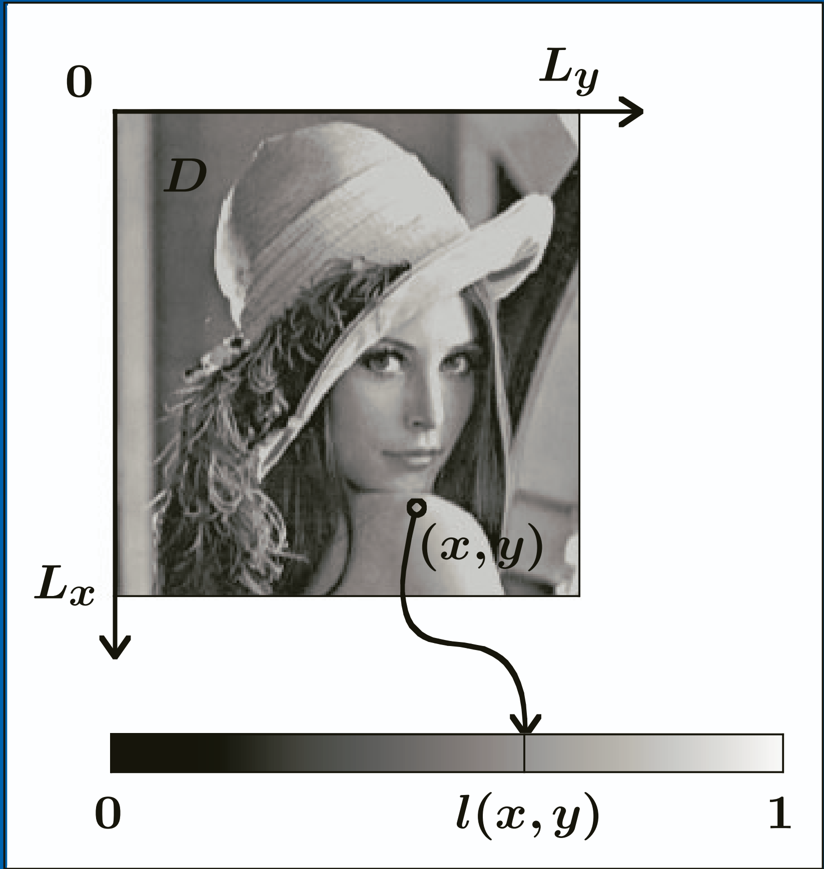
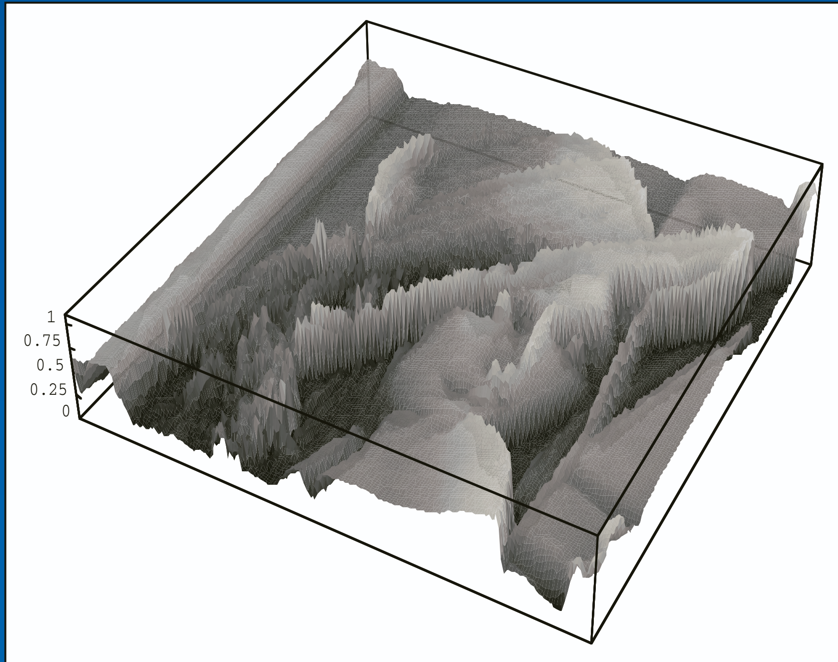
IMMAGINI CONTINUE

$D = [0, L_x] \times [0, L_y]$ (dominio dello spazio)
$l : D \to [0, 1]$ (funzione continua)



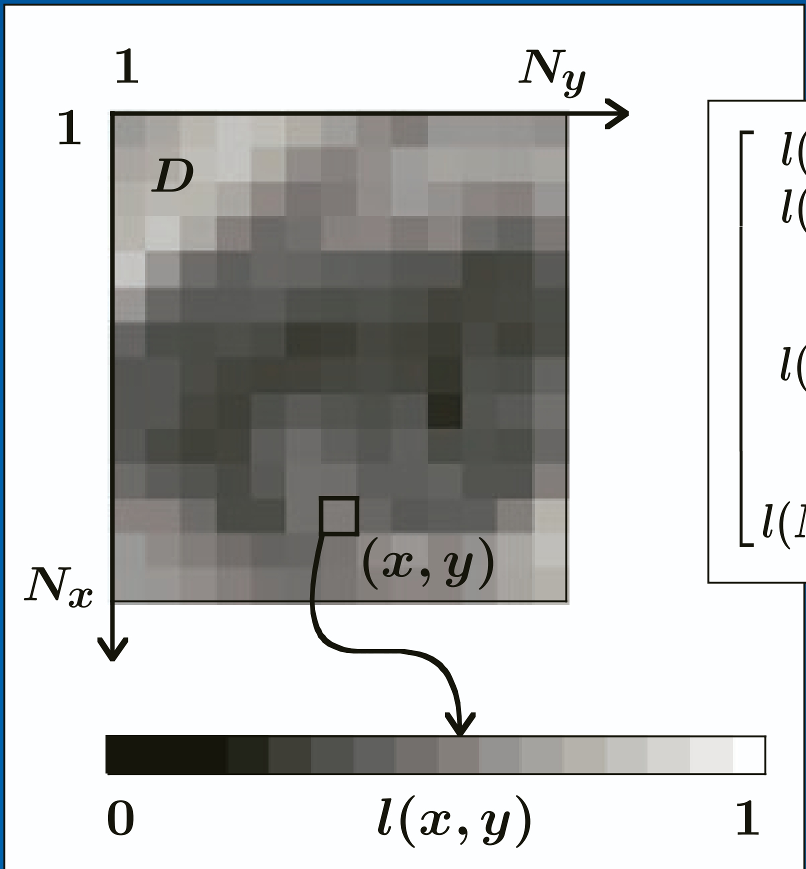


$$(l(x, y))_{\substack{0 \le x \le L_x \\ 0 \le y \le L_y}}$$

IMMAGINI DISCRETE

$D = \{1, \ldots, N_x\} \times \{1, \ldots, N_y\}$ (dominio dello spazio)

$l : D \rightarrow \{l_1, l_2, \ldots, l_N\} \subset [0, 1]$ (N livelli di grigio)



$$\begin{bmatrix} l(1,1) & l(1,2) & \cdots & l(1,y) & \cdots & l(1,N_y) \\ l(2,1) & l(2,2) & \cdots & l(2,y) & \cdots & l(2,N_y) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ l(x,1) & l(x,2) & \cdots & l(x,y) & \cdots & l(x,N_y) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ l(N_x,1) & l(N_x,2) & \cdots & l(N_x,y) & \cdots & l(N_x,N_y) \end{bmatrix}$$

$$(l(x, y))_{\substack{x=1,\ldots,N_x \\ y=1,\ldots,N_y}}$$

OPERATORI

$$(l(x,y))_{\substack{x=1,\dots,N_x\\ y=1,\dots,N_y}} \xrightarrow{H} (l^H(\bar{x},\bar{y}))_{\substack{\bar{x}=1,\dots,N_{\bar{x}}\\ \bar{y}=1,\dots,N_{\bar{y}}}}$$

$$l^H(x,y) = f_{x,y}(l(u,v)_{u,v}) \text{ con } f_{x,y} : [0,1]^{N_x N_y} \rightarrow [0,1]$$

$$l^H(x,y) = f_{x,y}(l(x+h,y+k)_{h,k}) \text{ con } h = u-x, k = v-y$$

INVARIANZA (per traslazioni): $f_{x,y} = f$ **per ogni** x, y

$$l^H(x,y) = f(l(x+h,y+k)_{h,k})$$

LINEARITÀ: $f_{x,y}$ **lineare per ogni** x, y

$$l^H(x,y) = \Sigma_{h,k} m(x,y,h,k) l(x+h,y+k)$$

OPERATORI PUNTUALI
(invarianti)

$$l^H(x,y) = f(l(x,y))$$
con $f : [0,1] \to [0,1]$

LUMINOSITÀ

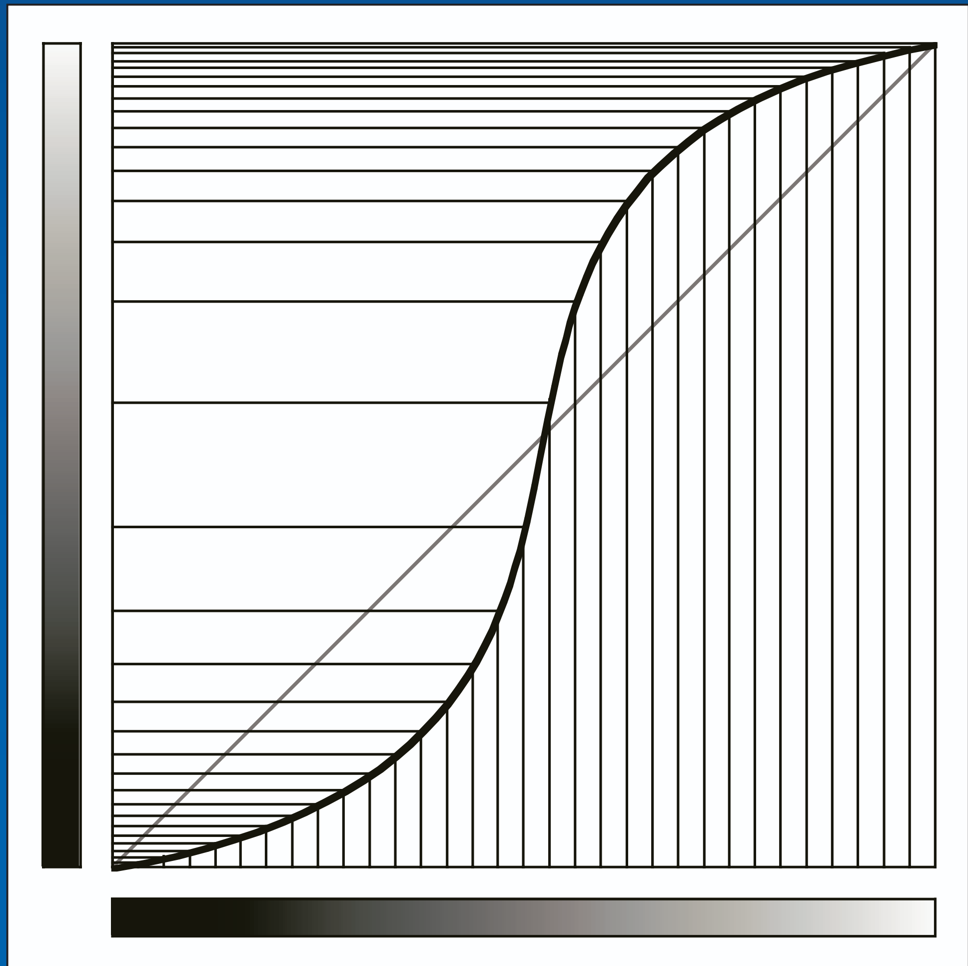
$f(l) > l \quad \longrightarrow$ **più**

$f(l) < l \quad \longrightarrow$ **meno**

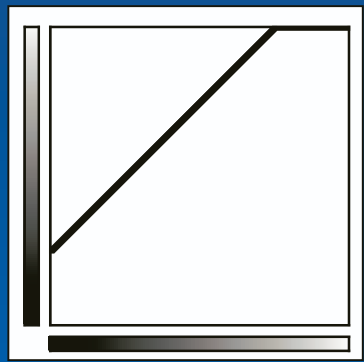
CONTRASTO

$f'(l) > 1 \quad \longrightarrow$ **più**

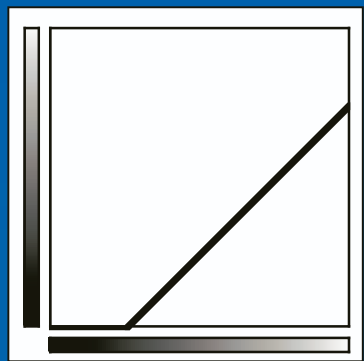
$f'(l) < 1 \quad \longrightarrow$ **meno**

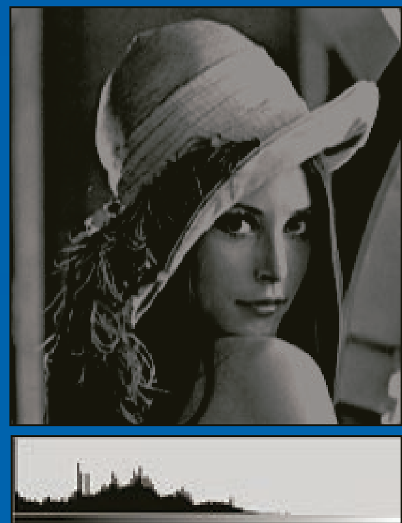


LUMINOSITÀ

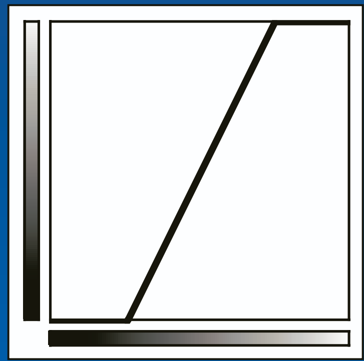


$$f(l) = l \pm c$$

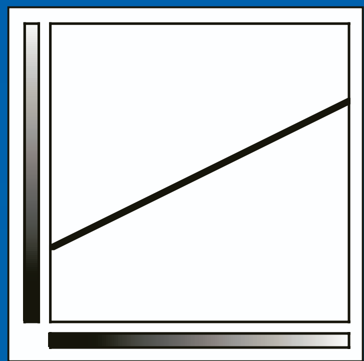


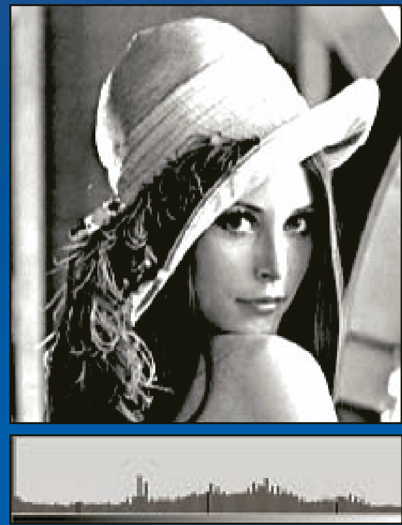


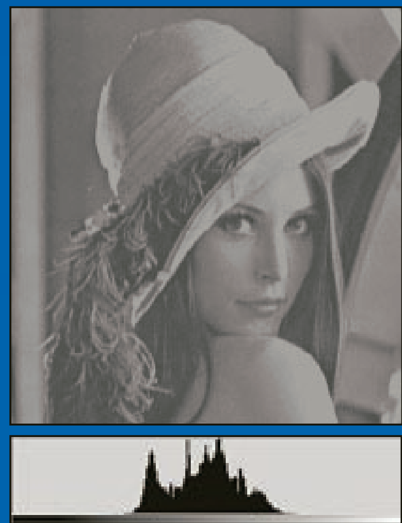
CONTRASTO



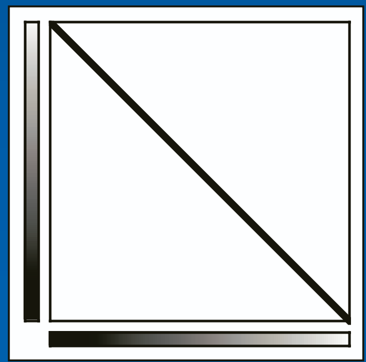
$$f(l) = m\,l \pm c$$





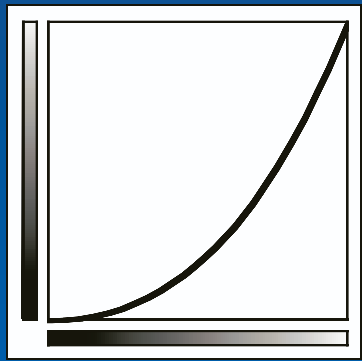


INVERSIONE

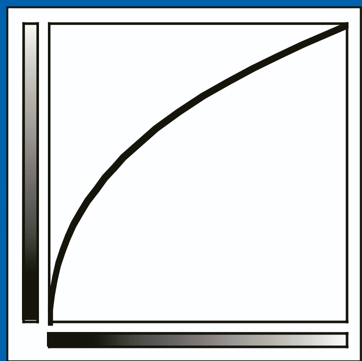


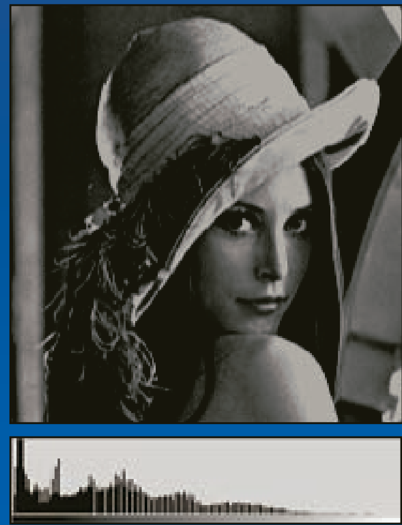
$$f(l) = 1 - l$$

GAMMA



$$f(l) = l^{\gamma}$$



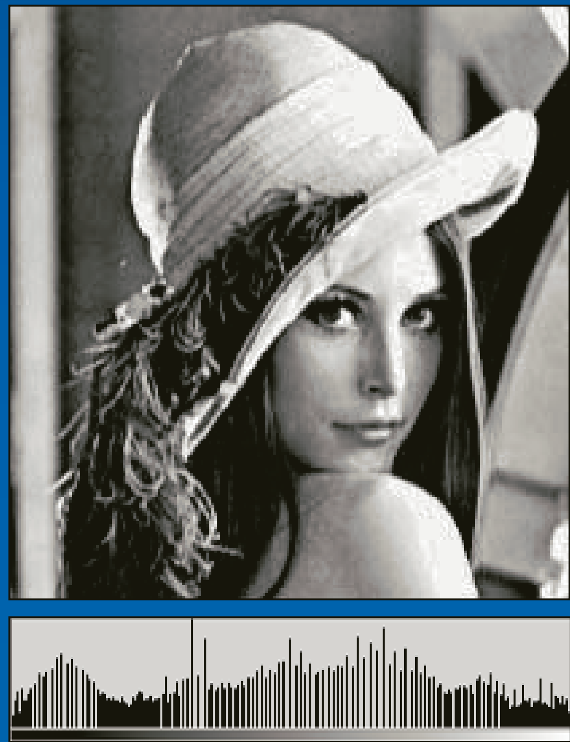


EQUALIZZAZIONE

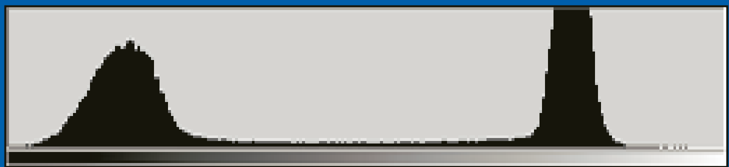
densità di probabilità uniforme

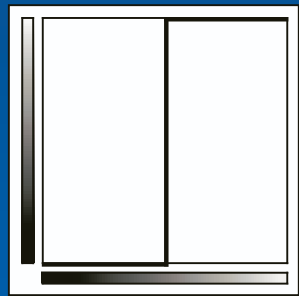
$$f(l) = \sum_{l_i \leq l} p(l_i)$$

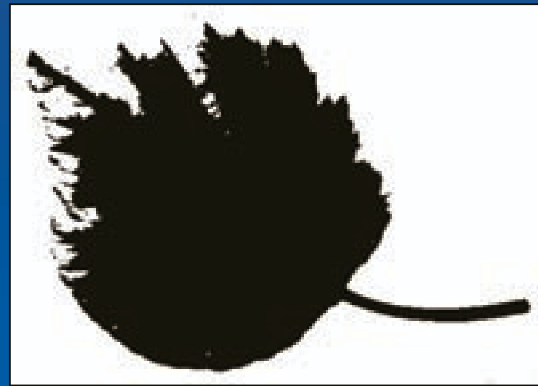
$$p(l_i) = \frac{n(l_i)}{N_x N_y}$$

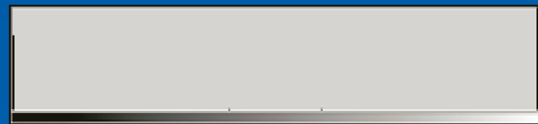


SOGLIE (riduzione livelli)

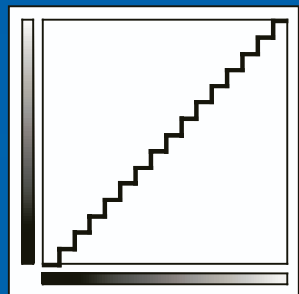








$f(l) = l_i$ **più vicino ad** l

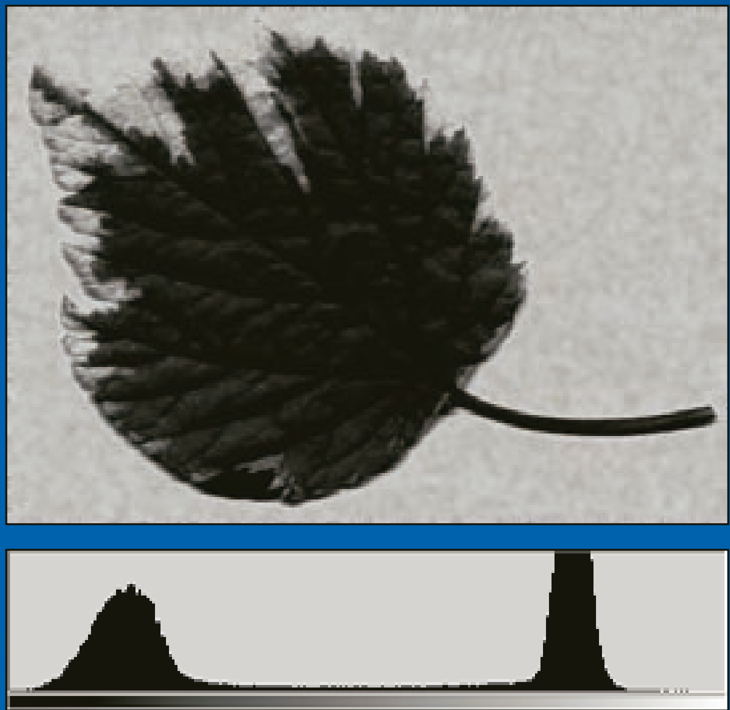


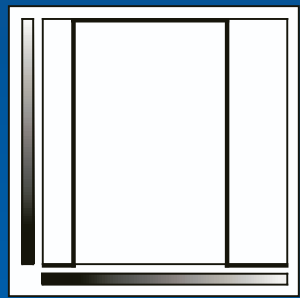
$f(l)_{\mathbf{bin}} = [l_{\mathbf{bin}}]_i$

$([\ ]_i =$ tronc. all'i-esimo bit$)$

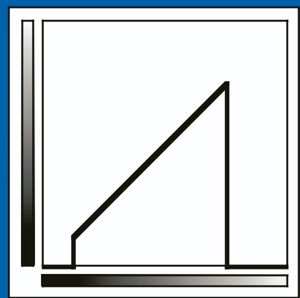
SOGLIE (thresholding)

$$f(l) = \begin{cases} 1 & \text{se } l_1 \le l \le l_2 \\ 0 & \text{altrimenti} \end{cases}$$

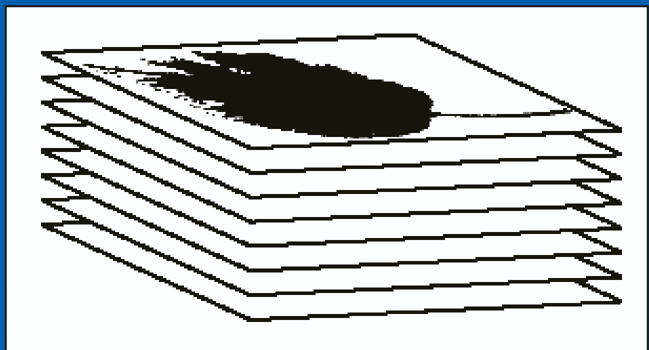




$$f(l) = \begin{cases} l & \text{se } l_1 \le l \le l_2 \\ 0 & \text{altrimenti} \end{cases}$$

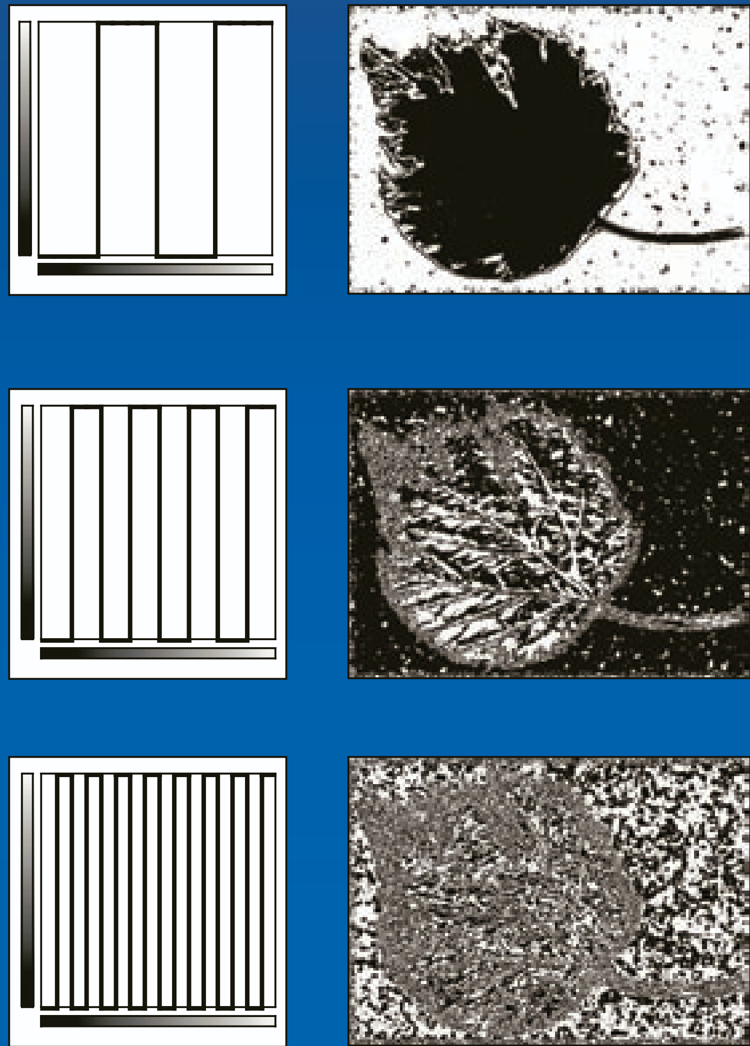


PIANI DI BIT



$$f_i(l_{\text{bin}}) = (\delta_{ij})_j \wedge l_{\text{bin}}$$

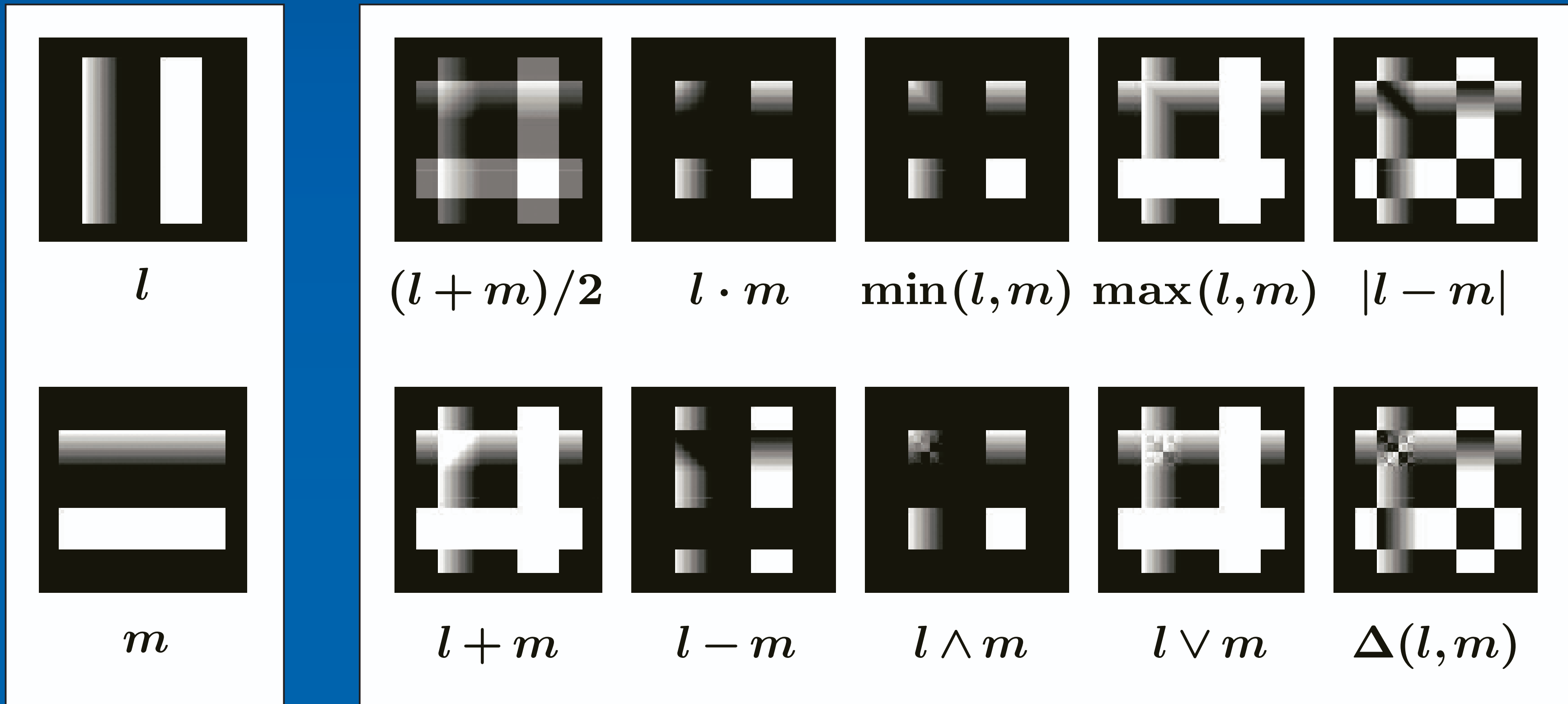
(i-esimo piano di bit)



FUSIONE

$$(l(x,y))_{x,y} \bullet (m(x,y))_{x,y} = (l(x,y) \bullet m(x,y))_{x,y}$$

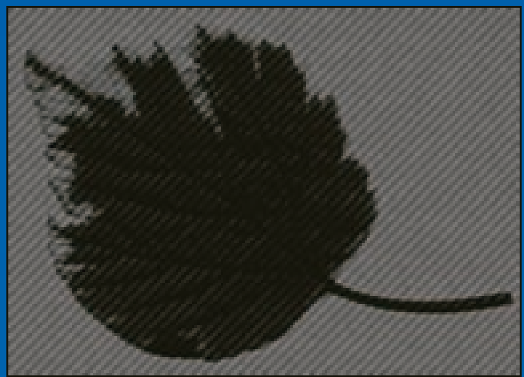
$$f_{x,y}(l) = l \bullet m(x,y) \quad \text{(operatore puntuale non invariante)}$$



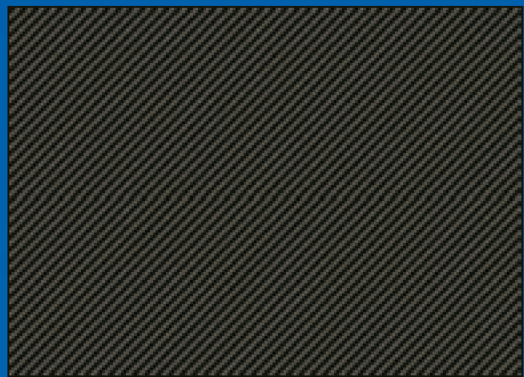
FUSIONE ADDITIVA

immagini con rumore

immagine mediata



originale (l)



disturbo (m)

$m - l$

FUSIONE MOLTIPLICATIVA

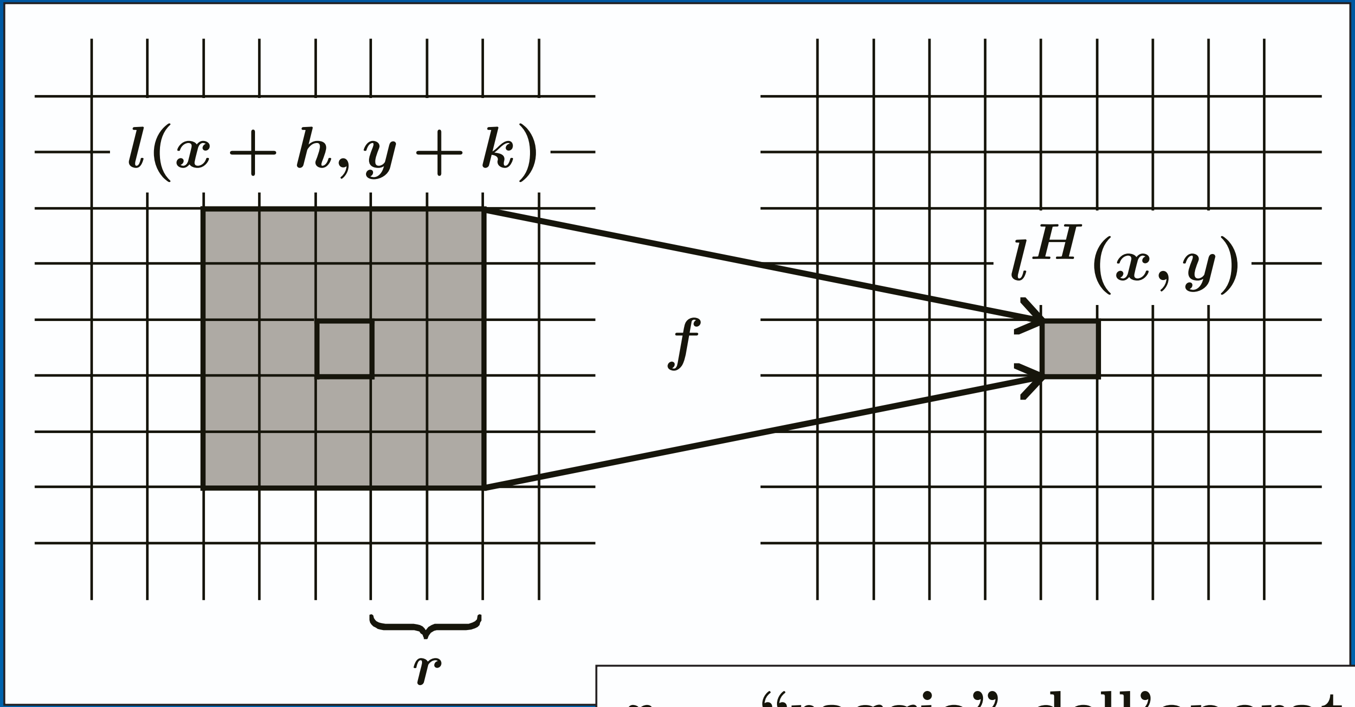
immagine originale (l)

luce non uniforme (m)

immagine corretta (l/m)

OPERATORI LOCALI (invarianti)

$$l^H(x,y) = f(l(x+h,y+k)_{-r\leq h,k\leq r})$$



r = "raggio" dell'operatore

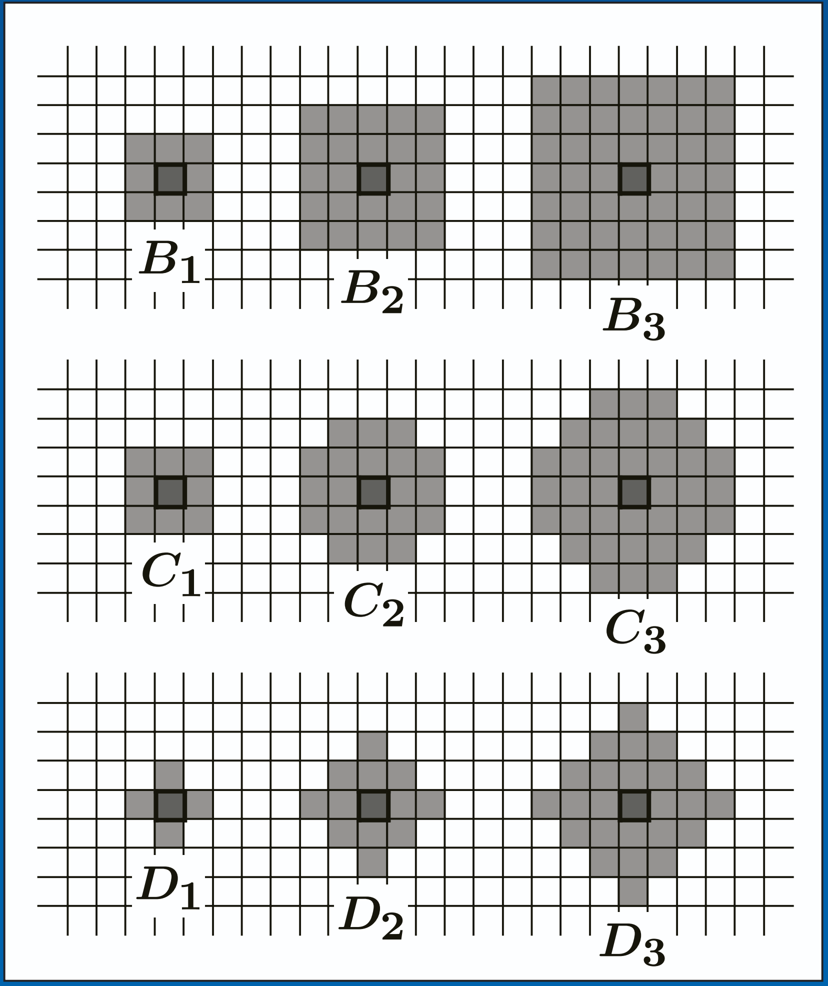
METRICHE E INTORNI

$$d((x_1, y_1), (x_2, y_2))$$

$$\max(|x_1 - x_2|, |y_1 - y_2|)$$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$|x_1 - x_2| + |y_1 - y_2|$$



OPERATORI LOCALI LINEARI

$$l^H(x,y) = \Sigma_{-r \leq h,k \leq r} m_H(h,k) l(x+h, y+k)$$

MATRICE di H: $(m_H(h,k))_{h,k}$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$l = (\delta_{(0,0),(x,y)})_{x,y}$

4	3	2
5	9	1
6	7	8

$(m_H(h,k))_{h,k}$

0	0	0	0	0	0	0
0	0	8	7	6	0	0
0	0	1	9	5	0	0
0	0	2	3	4	0	0
0	0	0	0	0	0	0

$l^H = (\widetilde{m}_H(-x,-y))_{x,y}$

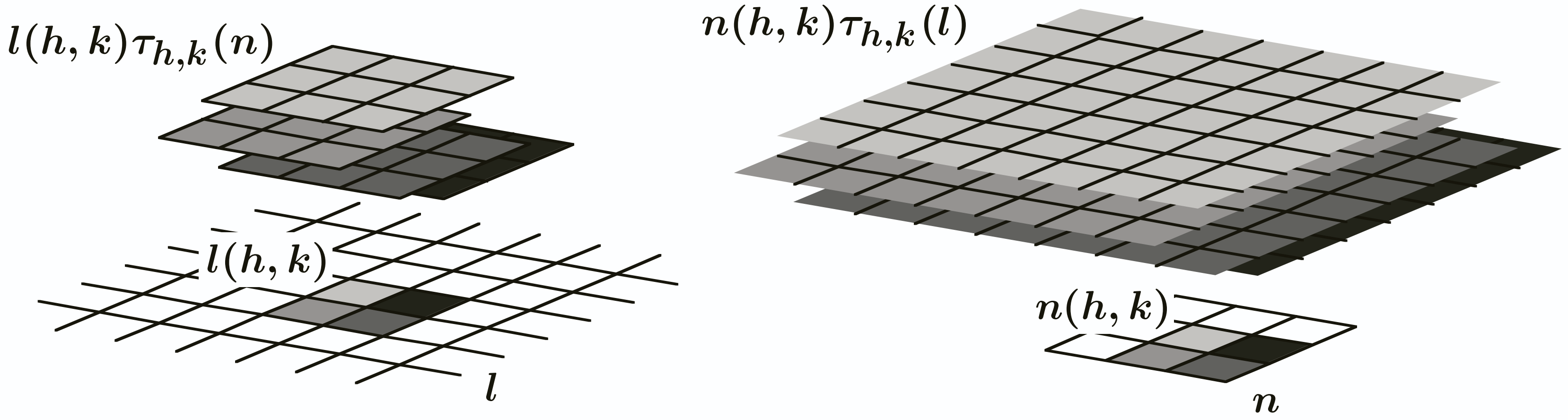
NUCLEO di H: $(n_H(x,y))_{x,y} = (\widetilde{m}_H(-x,-y))_{x,y}$

CONVOLUZIONE

$$(l(x,y))_{x,y} * (n(x,y))_{x,y} = \left(\Sigma_{\substack{x'+x''=x \\ y'+y''=y}} l(x',y')\, n(x'',y'')\right)_{x,y}$$

$$(l * n)(x,y) = \Sigma_{h,k} l(h,k) n(x-h, y-k) = \Sigma_{h,k} n(h,k) l(x-h, y-k)$$

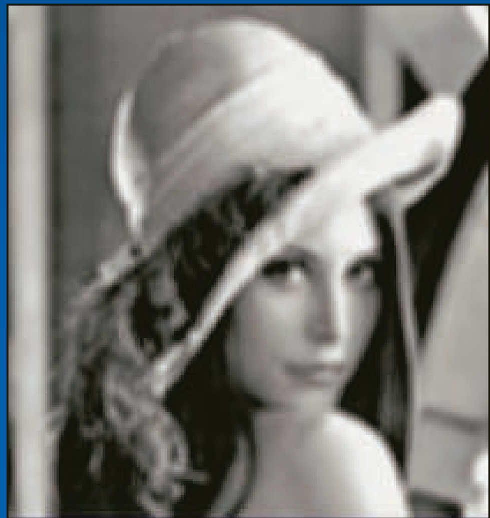
$$l * n = \Sigma_{h,k} l(h,k) \tau_{h,k}(n) \qquad l * n = \Sigma_{h,k} n(h,k) \tau_{h,k}(l)$$



$$l^H = l * n_H$$

FILTRI DI MEDIA

Riduzione del rumore e dei dettagli (smoothing)



$$m^H = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$m^H = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

FILTRI GAUSSIANI

Riduzione del rumore e dei dettagli (smoothing)

$$m^H = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$m^H = \frac{1}{256}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

FILTRI MEDIANI

Operatori locali NON LINEARI

$$l^H(x,y) = \text{mediana}(l(x+h, y+k))_{B_r/C_r/D_r}$$

Riduzione del rumore e dei dettagli

(senza sfurmare i contorni e i livelli di grigio)

$r = 1$

$r = 2$

FILTRI DI RAFFINAMENTO

Accentuazione dei dettagli e dei contorni (sharpening)

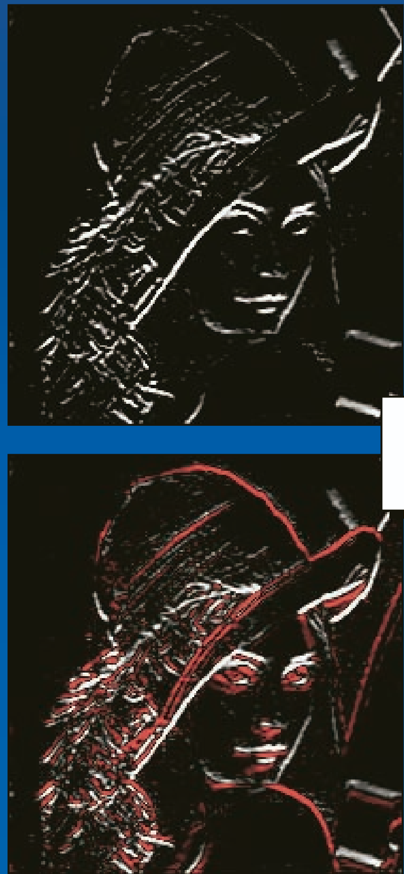
$w = 8$

$w = 12$

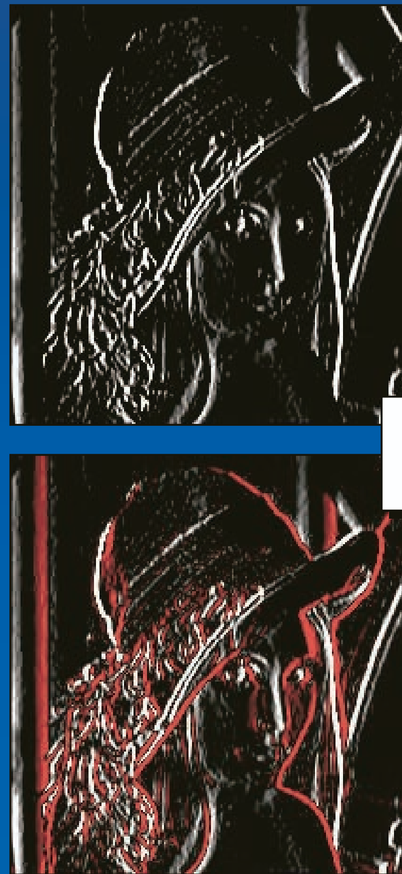
$$m^H = \begin{bmatrix} -1 & -1 & -1 \\ -1 & w & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

FILTRI GRADIENTE

$$\nabla l = (l'_x, l'_y)$$



l'_x

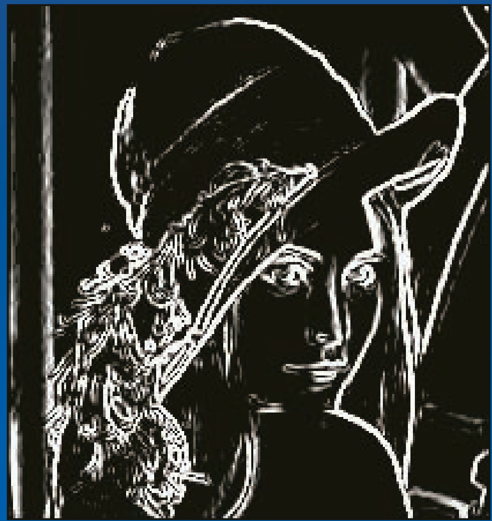


l'_y

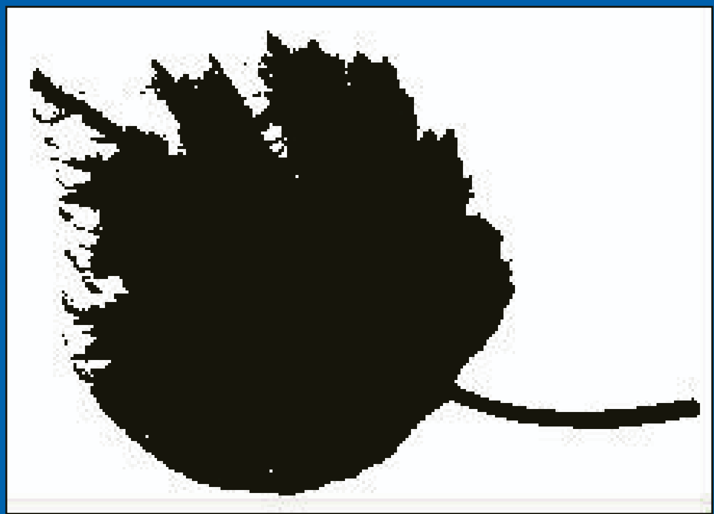
$$m^H = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$m^H = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

CONTORNI



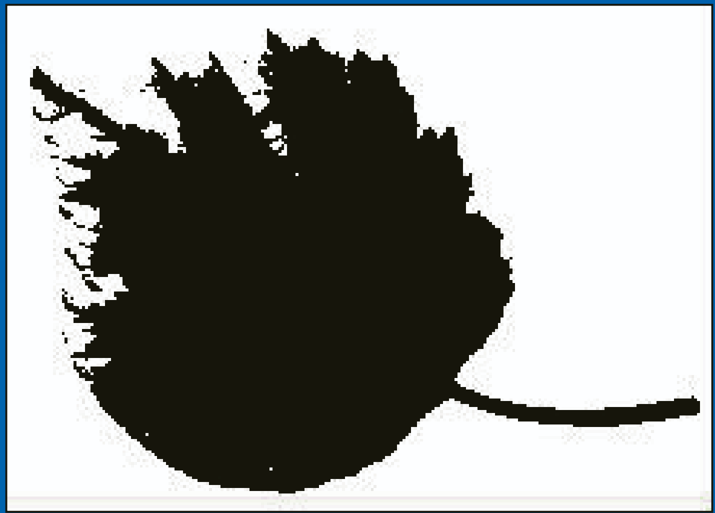
$$|\nabla l|^2 = {l'_x}^2 + {l'_y}^2$$

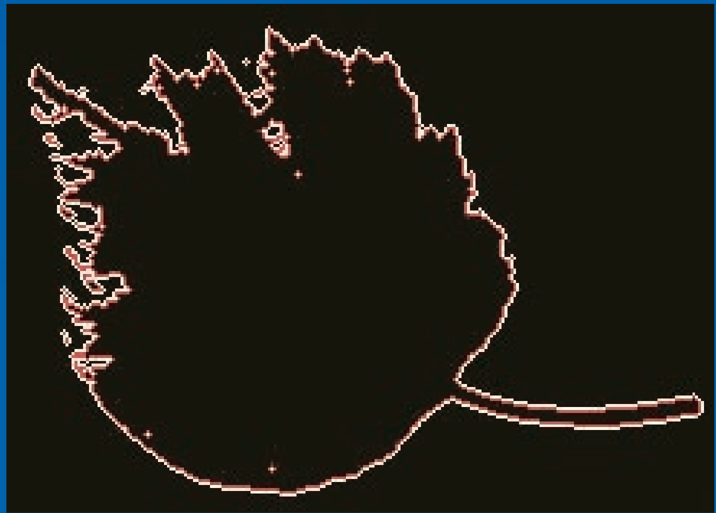


FILTRO LAPLACIANO

$$m^H = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\Delta l = l''_{xx} + l''_{yy}$$



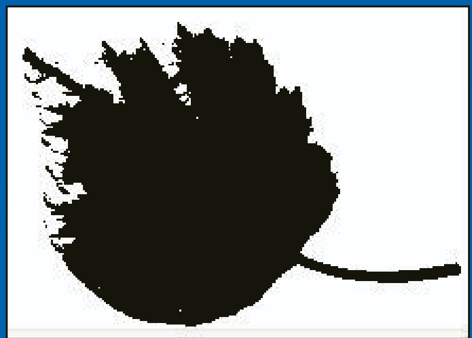
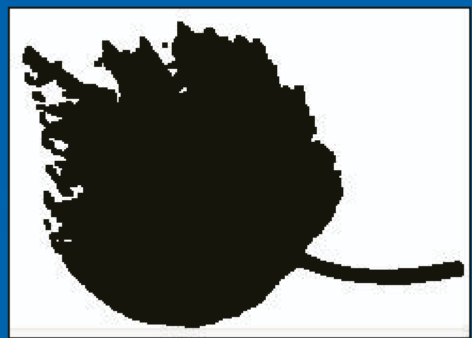
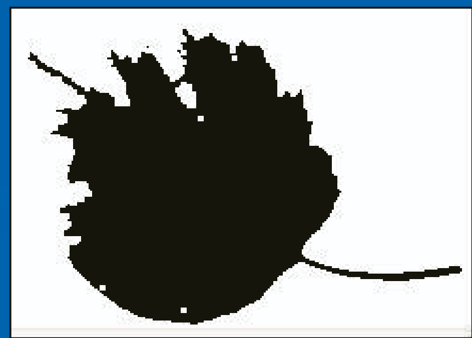


FILTRI DI MIN E MAX

Operatori locali NON LINEARI

$$l^H(x,y) = \min/\max(l(x+h, y+k))_{B_r/C_r/D_r}$$



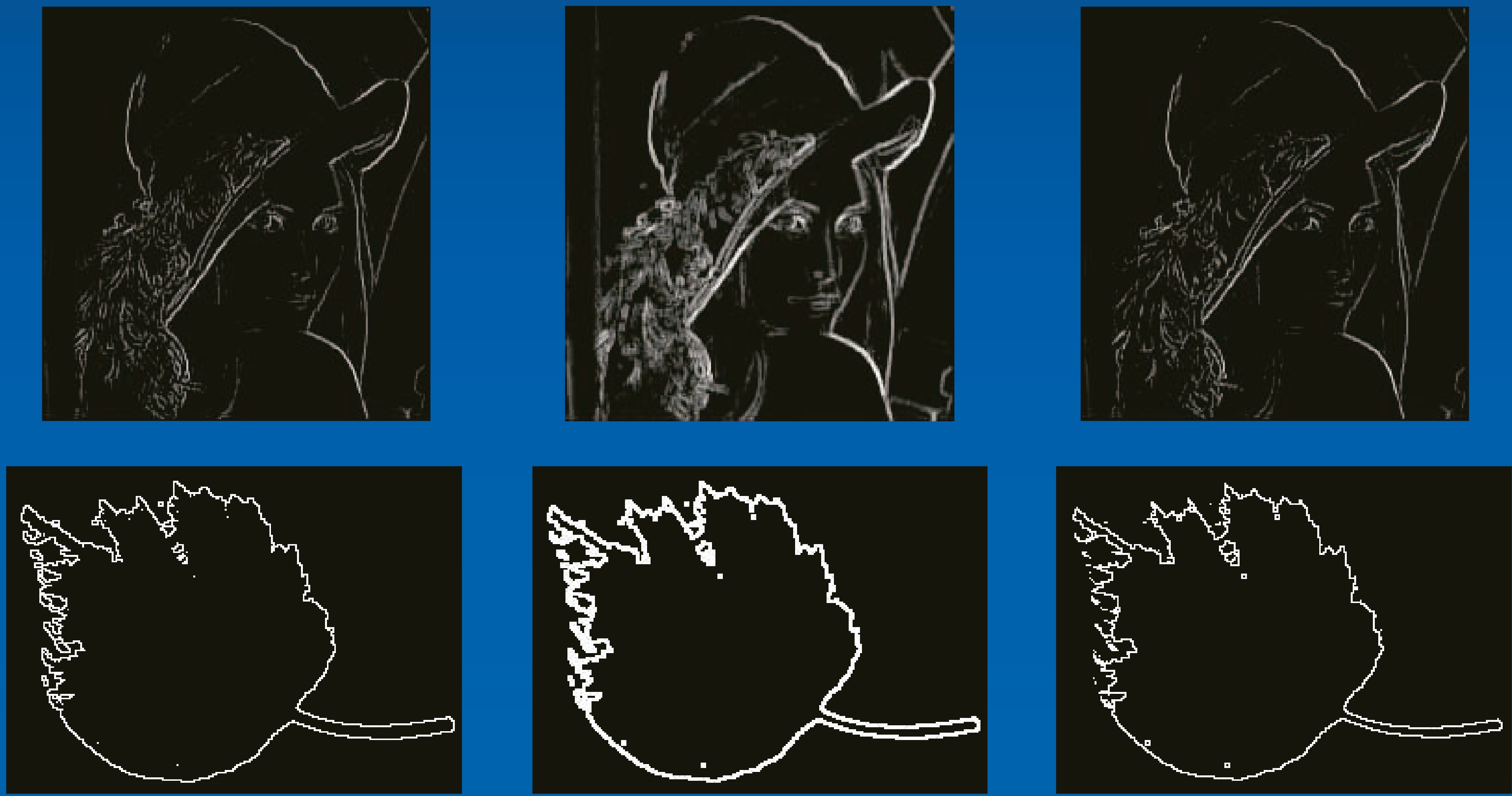




min $(r=1)$

max $(r=1)$

CONTORNI

$\max_1 l - l$ | $\max_1 l - \min_1 l$ | $l - \min_1 l$



lineamenti “esterni” | **lineamenti “interni”**