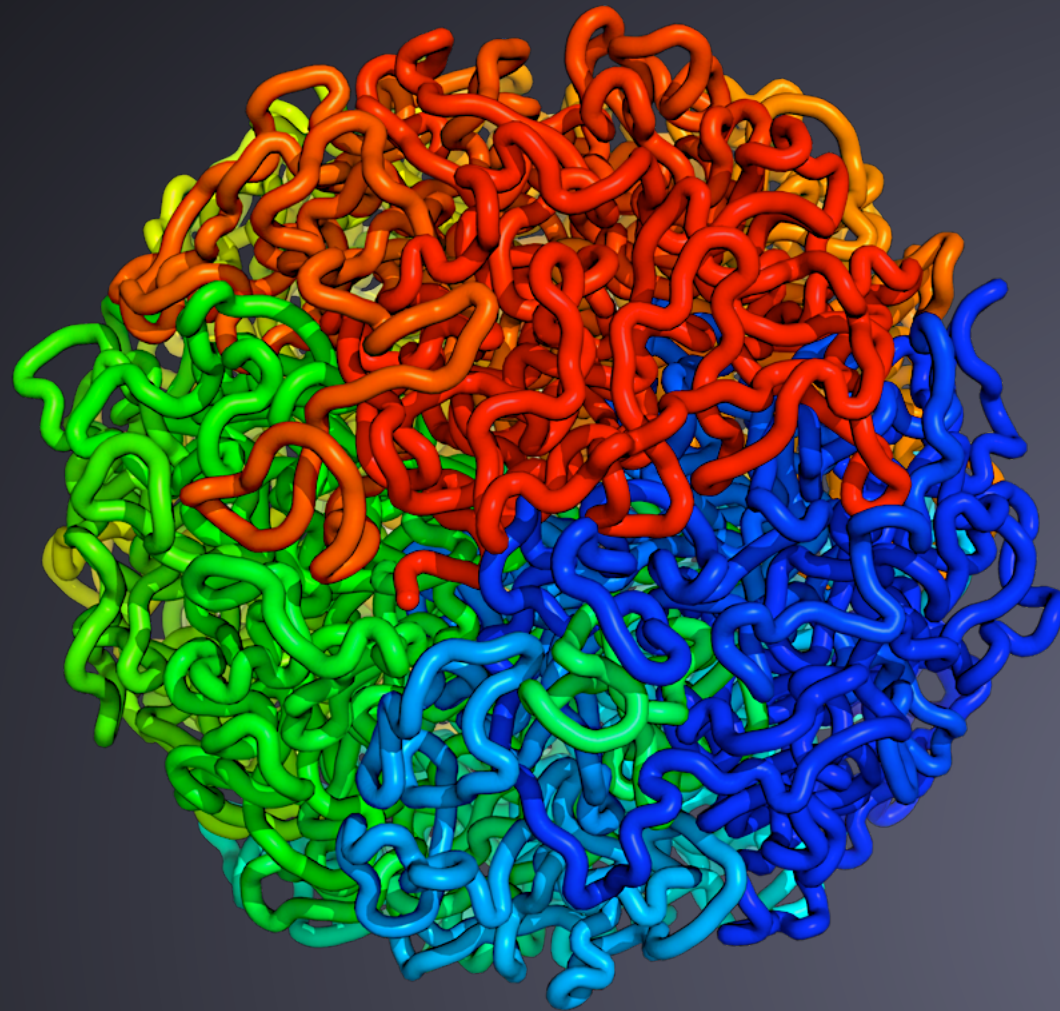


# La geometria del DNA

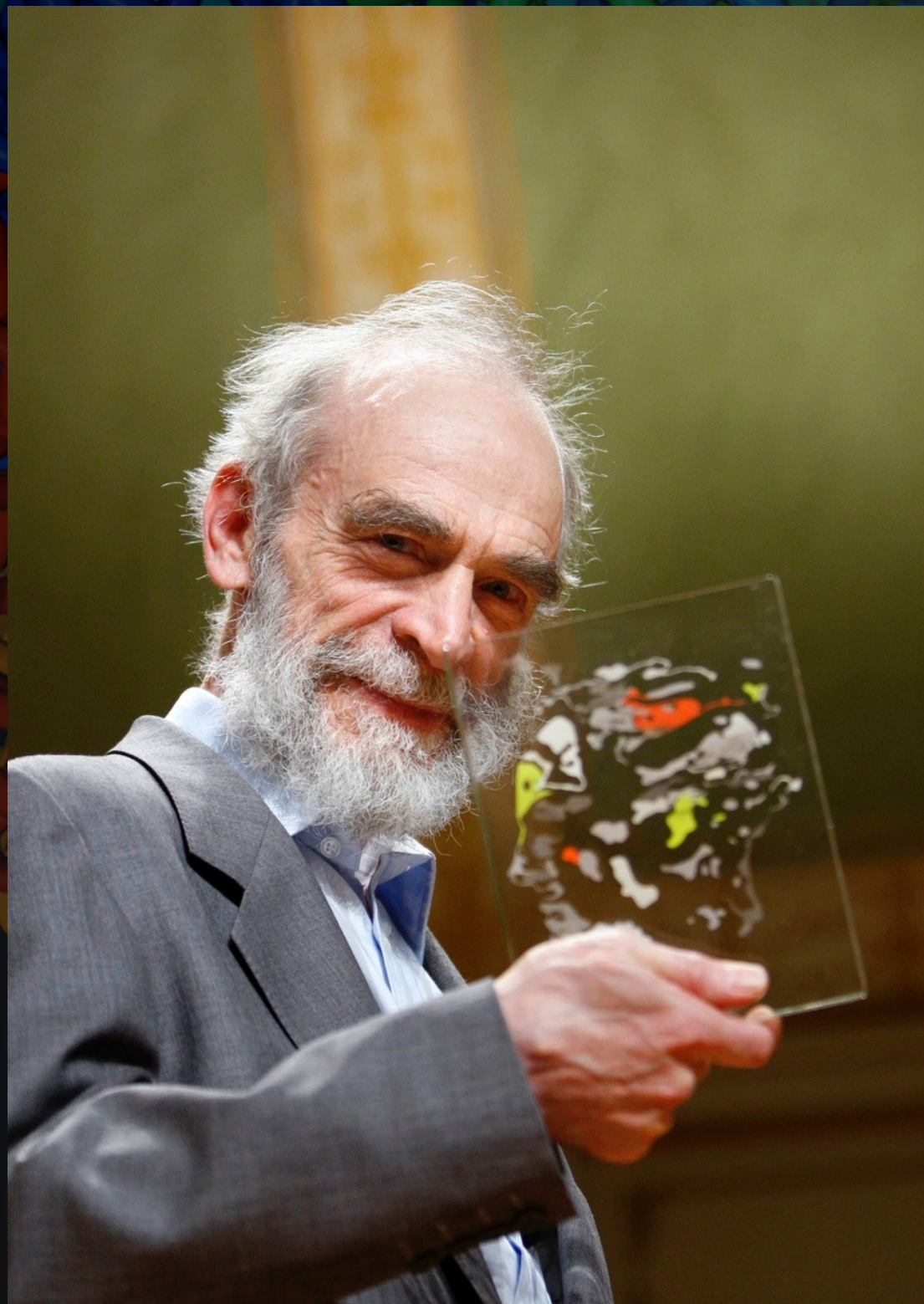
*Riccardo Piergallini*







# Biologia & matematica



Molti problemi nelle scienze sperimentali hanno a che fare con enormi quantità di dati debolmente strutturati. Tali problemi, in bilico tra simmetria e caos, attendono la nascita di una nuova matematica.

Spesso noi matematici non sappiamo cosa accade nelle scienze sperimentali e nell'ingegneria, mentre gli scienziati sperimentali e gli ingegneri non hanno idea delle opportunità offerte dalla matematica pura.

Occorre una nuova generazione di matematici capaci di mediare tra matematica pura e scienza applicata. La fertilizzazione incrociata delle idee è fondamentale per la progresso della matematica e della scienza.

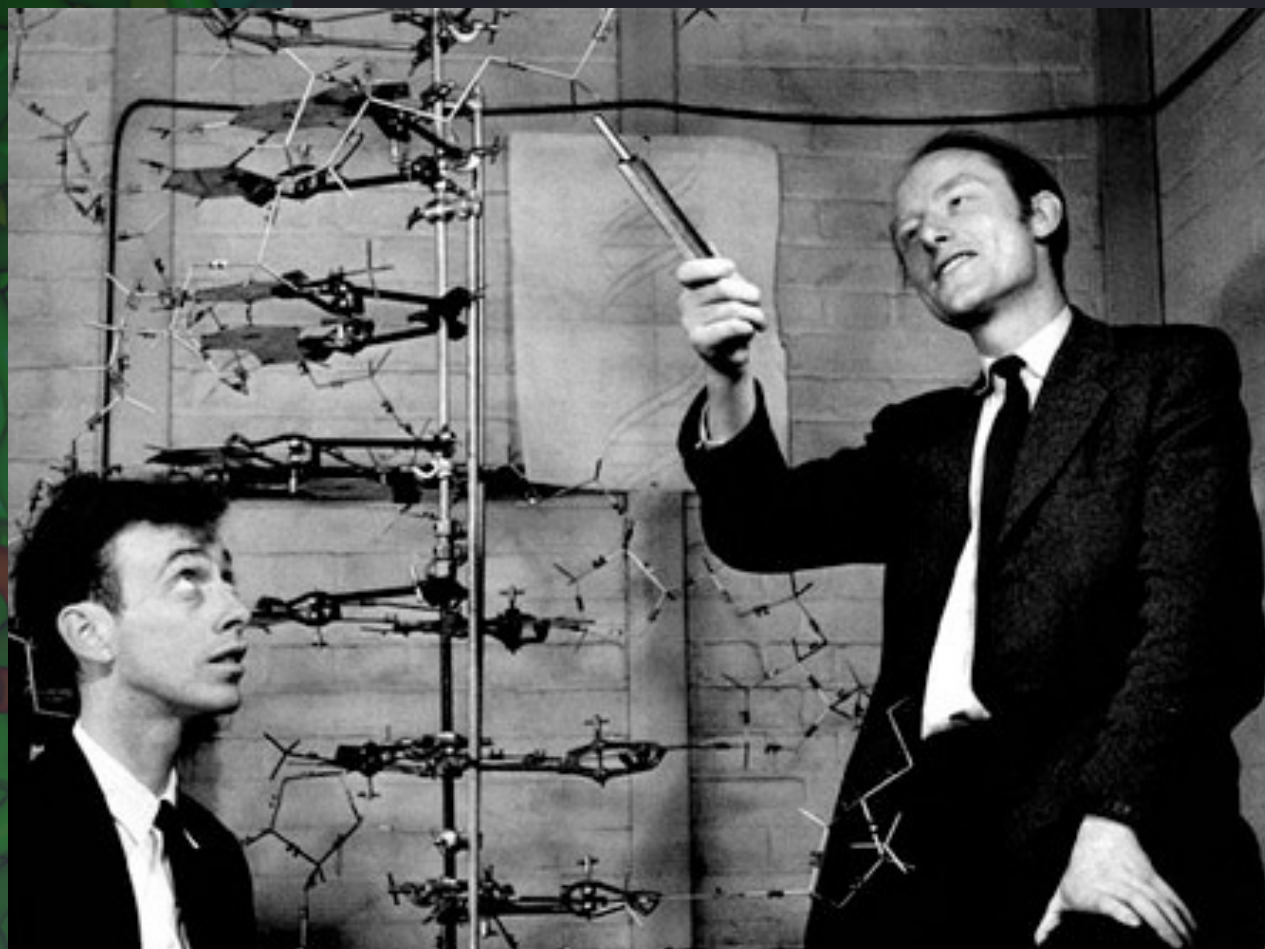
*Misha Gromov*





# Doppia elica

*Rosalind Franklin*

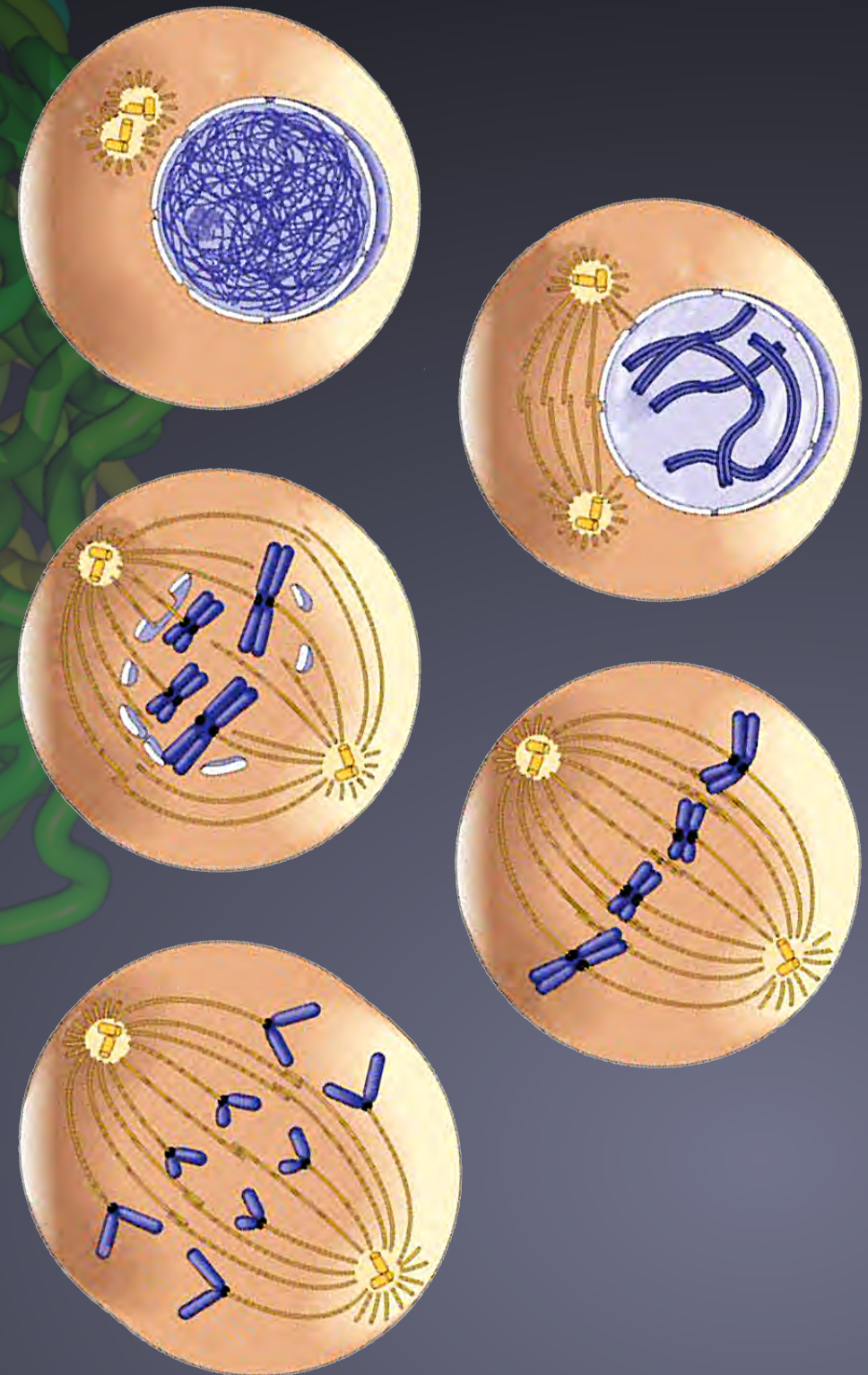
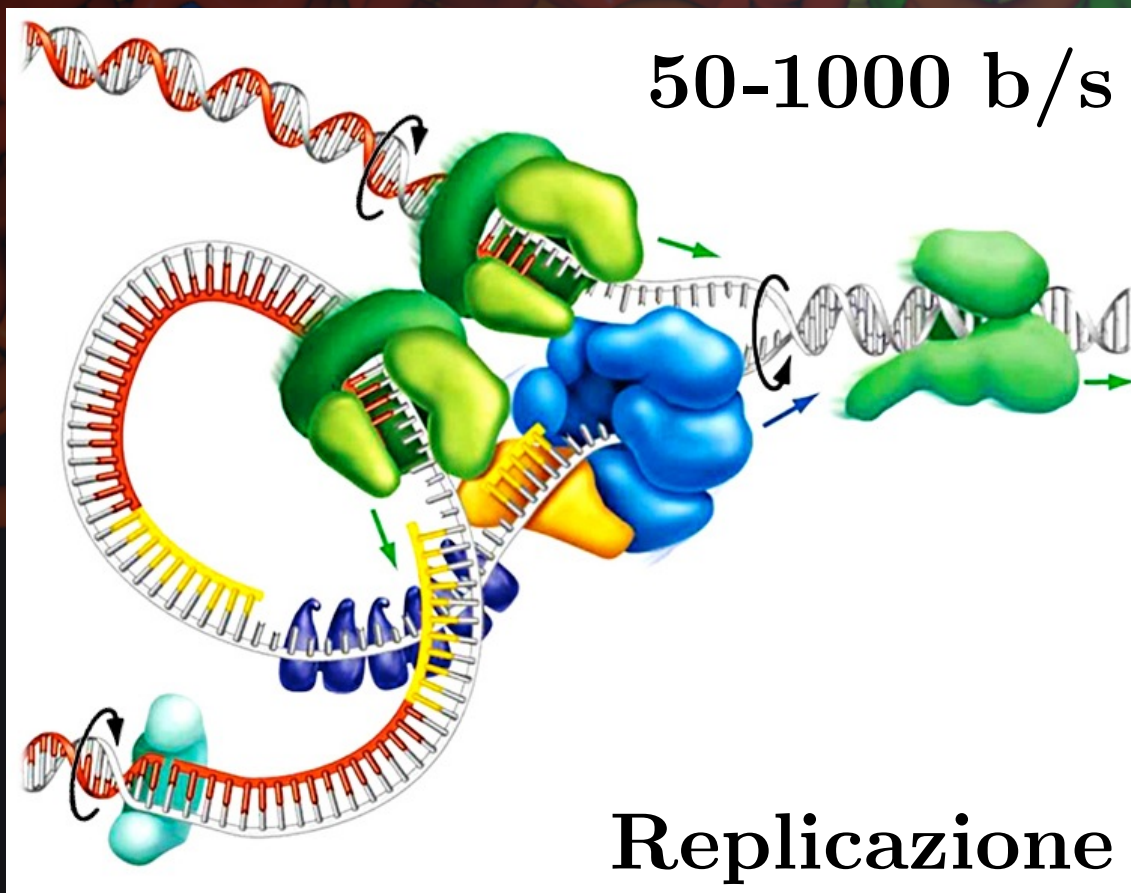
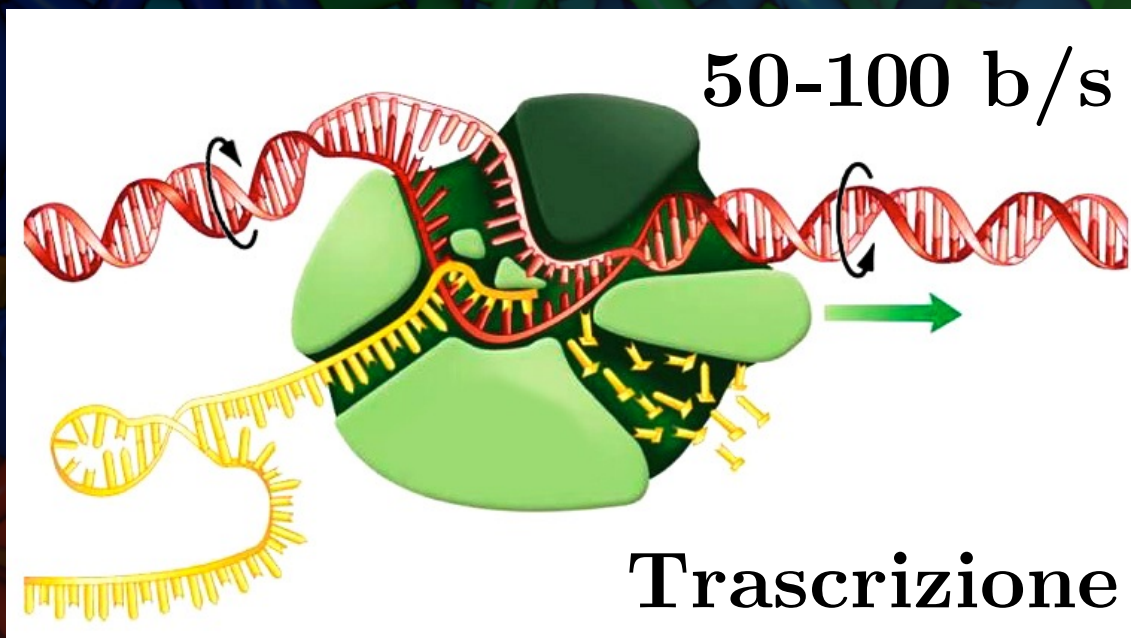


Le due catene del nostro modello si avvolgono tra loro, è quindi essenziale che possano districarsi per separarsi. Al momento è difficile immaginare come ciò possa avvenire senza che tutto si aggrovigli, ma tale obiezione non sarà insuperabile.

*Watson e Crick*

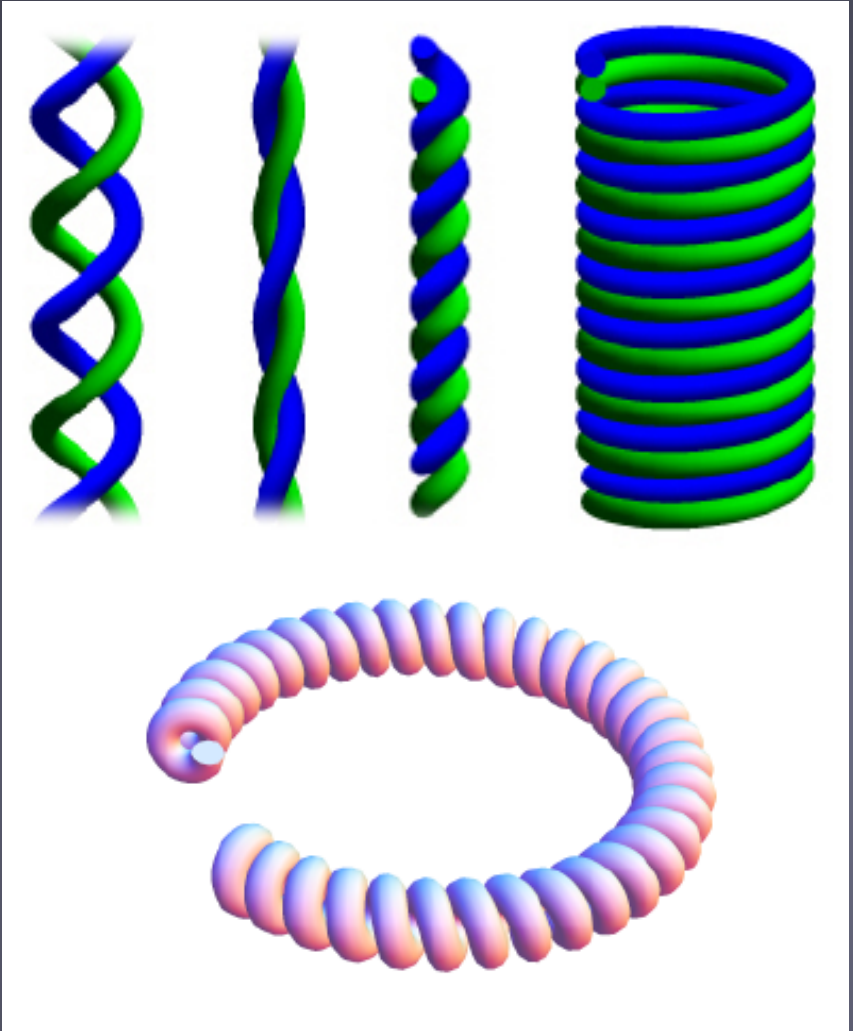
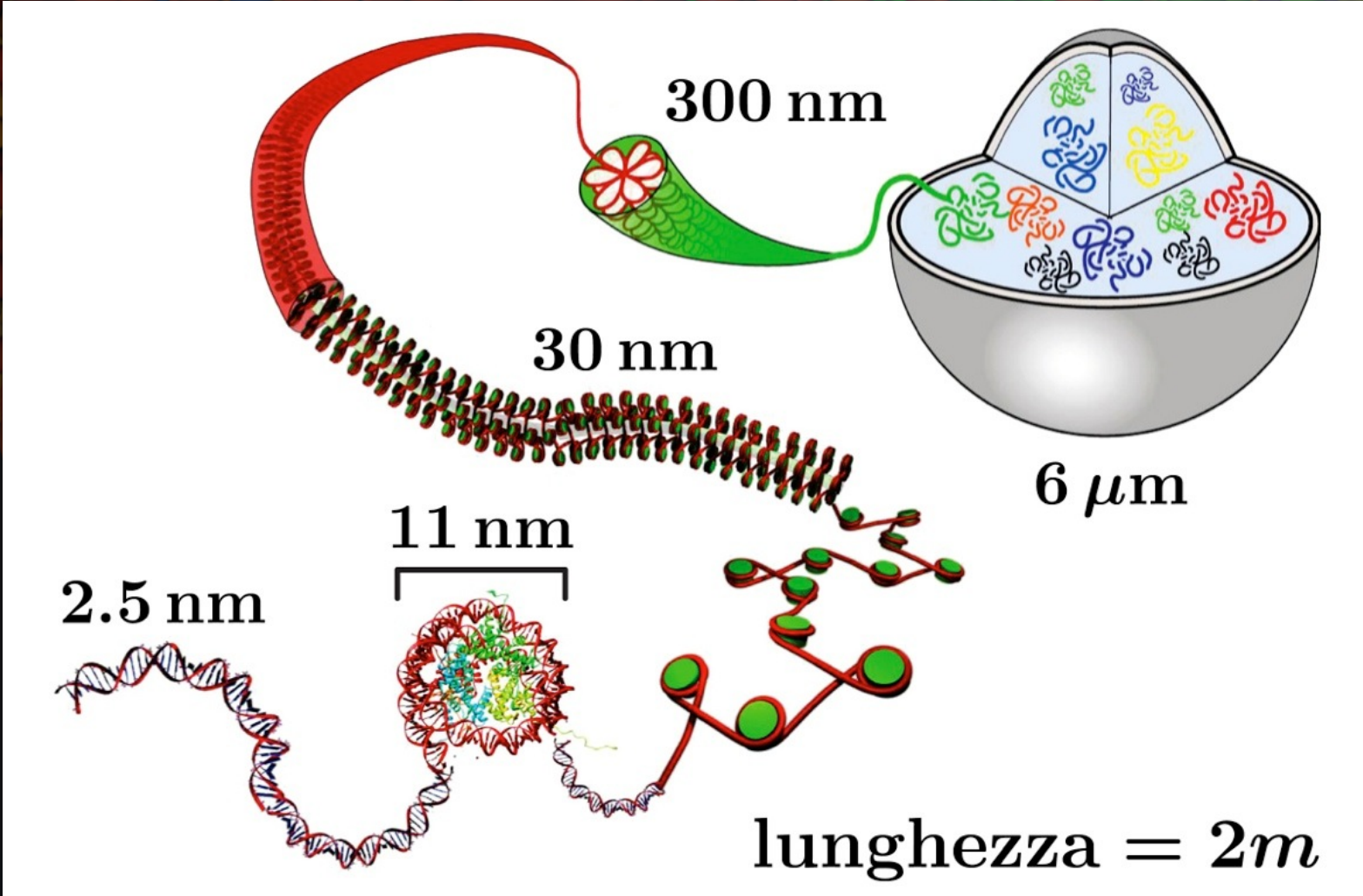
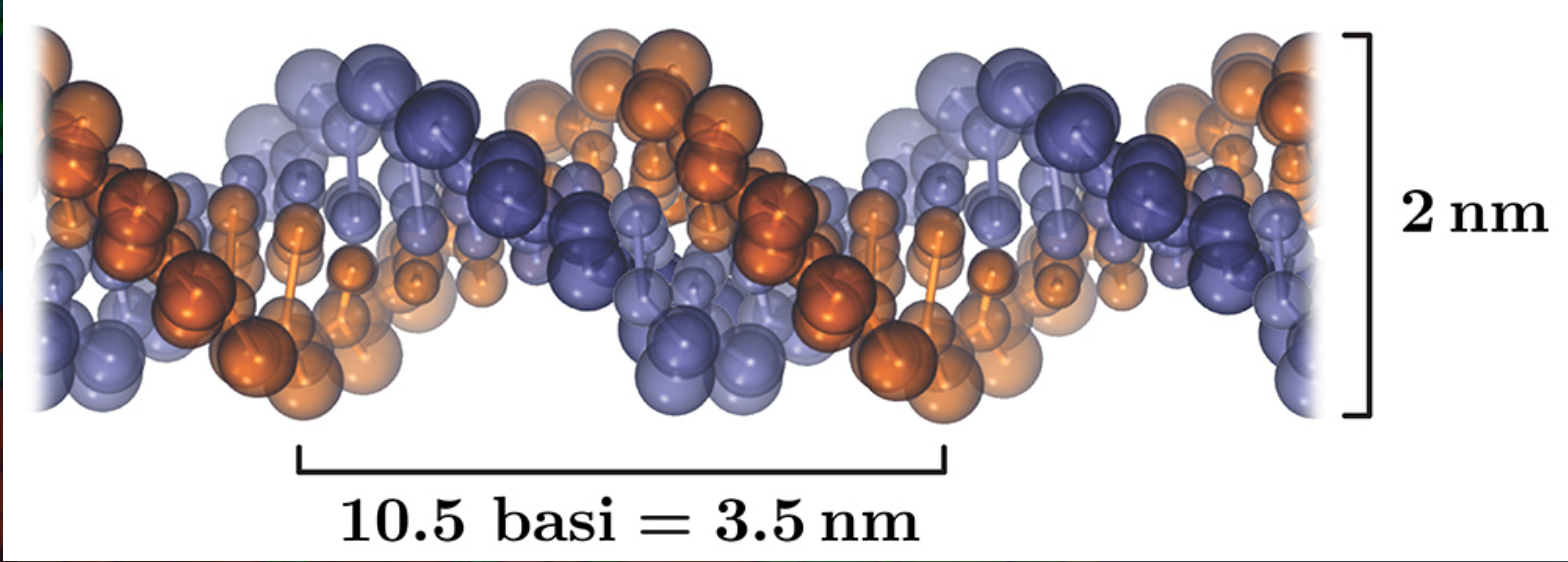


# Trascrizione e replicazione





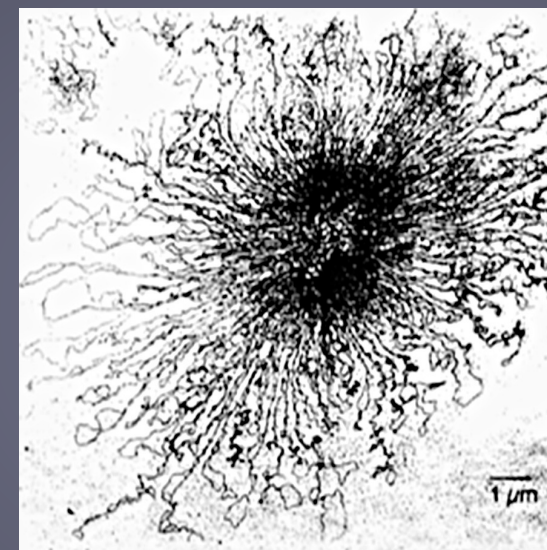
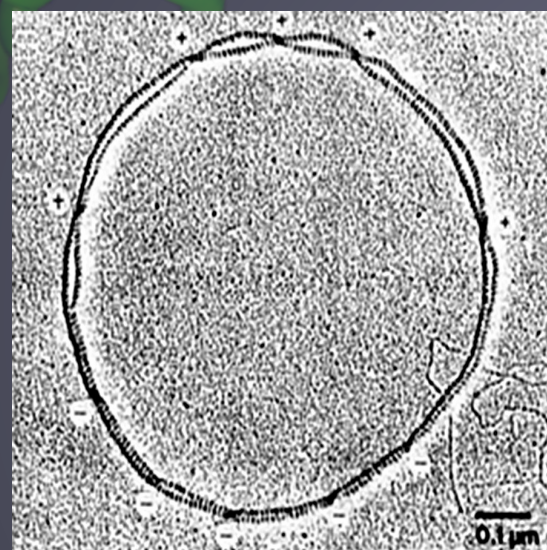
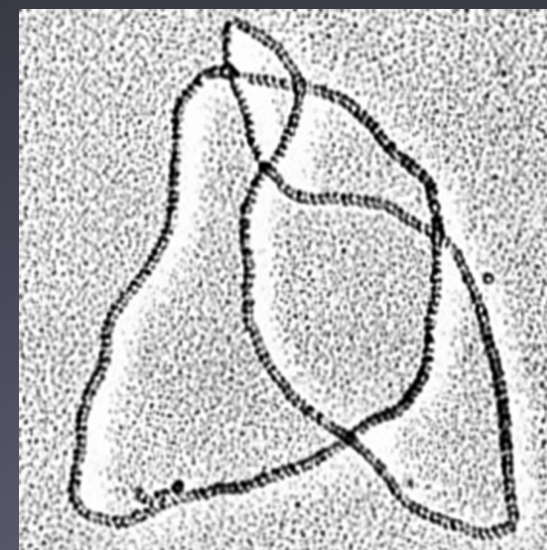
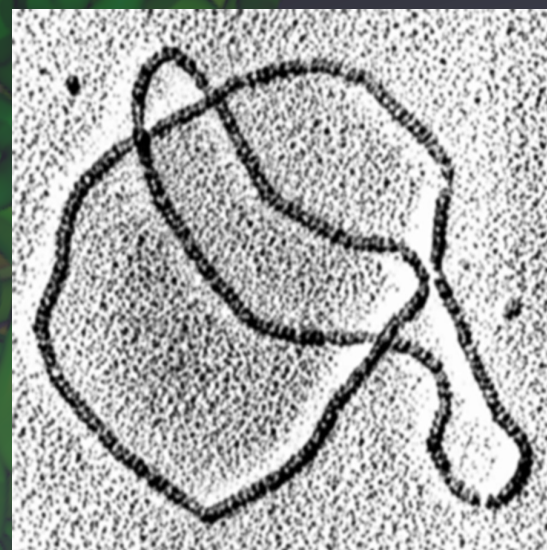
# Cromatina





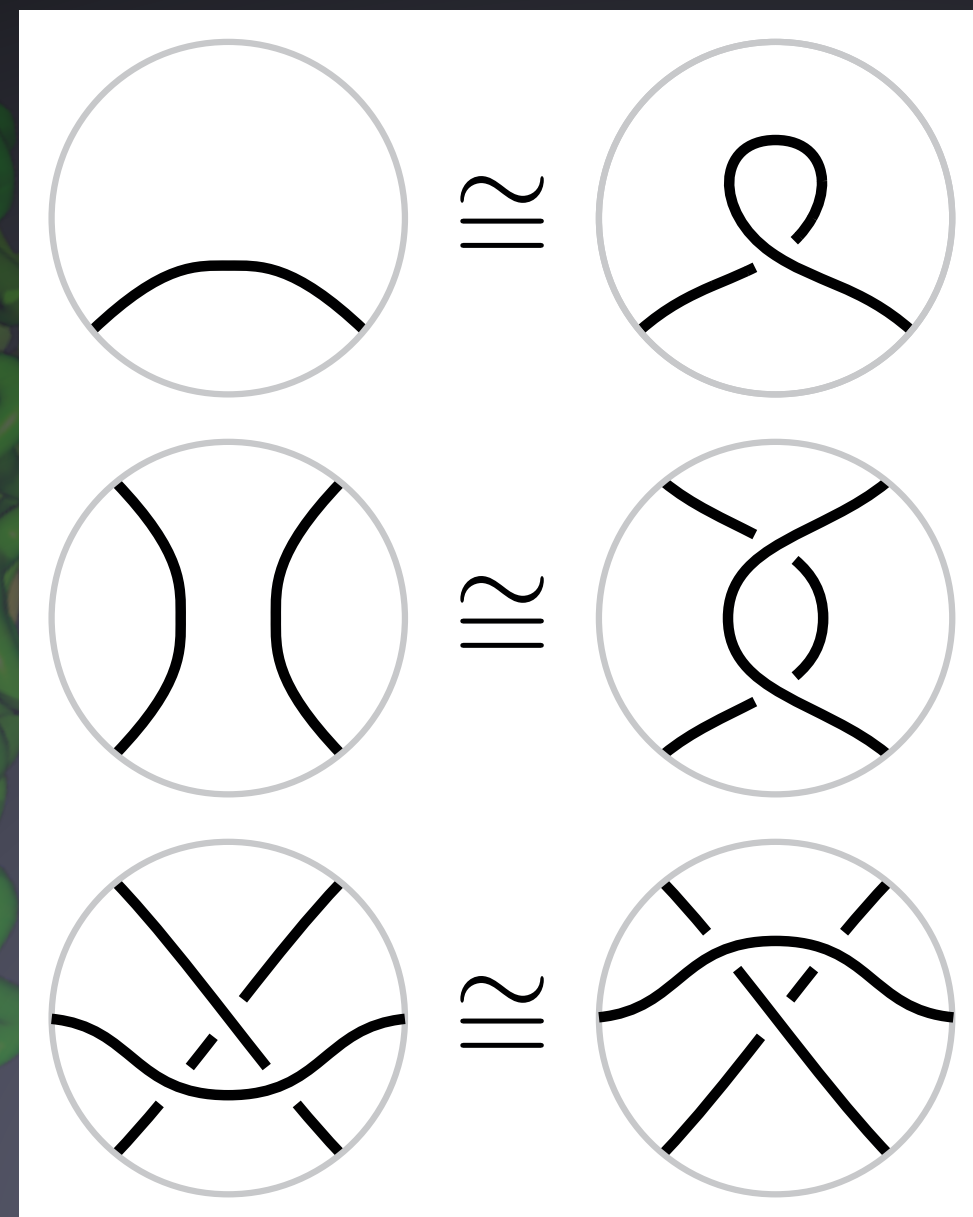
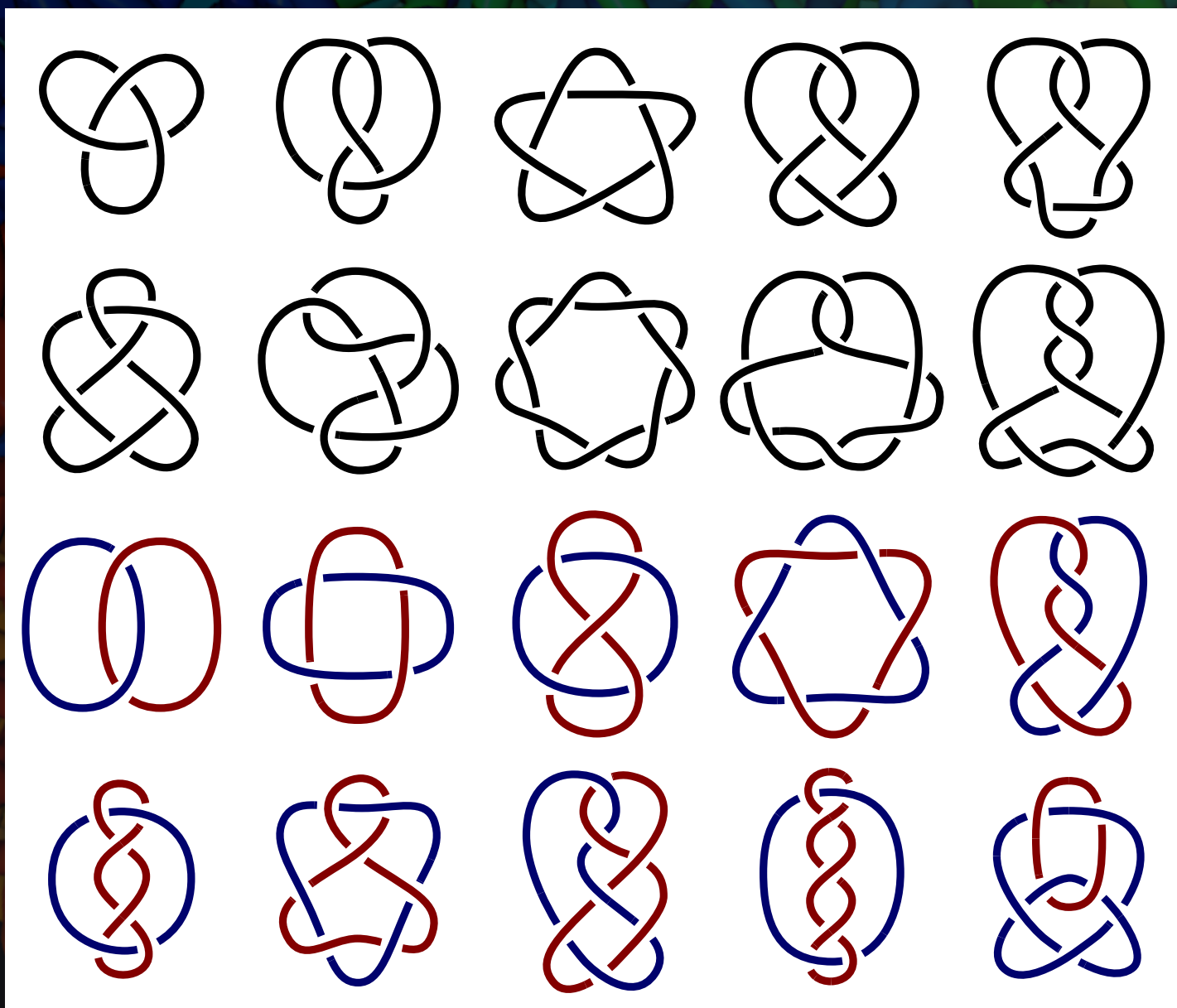
# Nodi topologici

*Curve chiuse nello spazio  
a meno di deformazioni continue  
(senza tagli o auto-attraversamenti)*





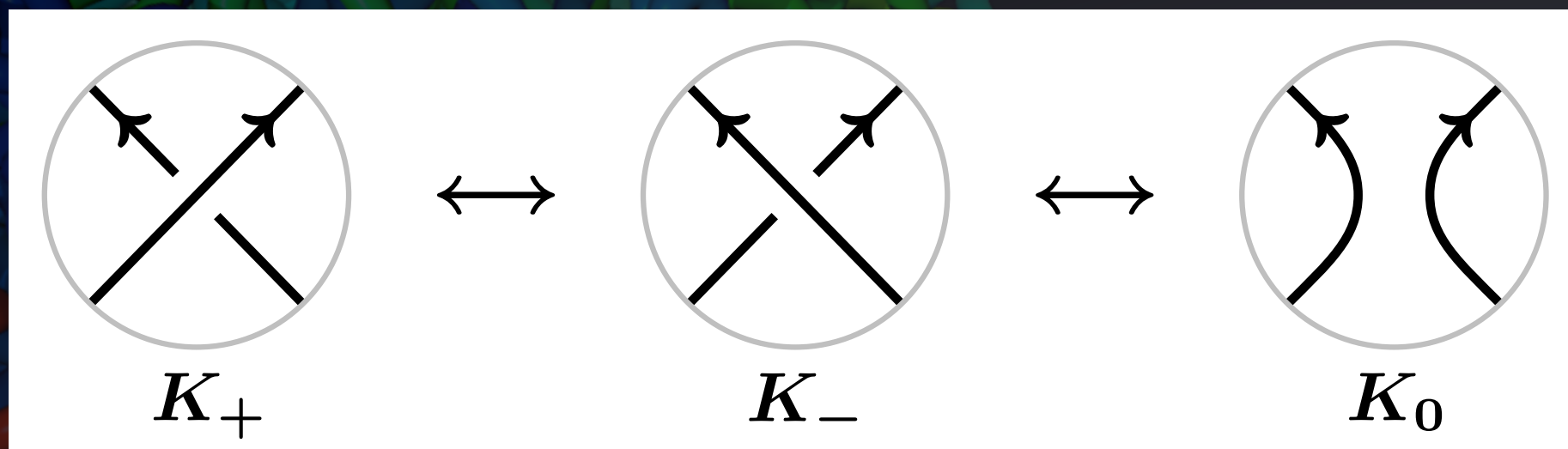
# Diagrammi e deformazioni



$c(K)$  = minimo numero di incroci per un diagramma di  $K$   
 $u(K)$  = minimo numero di incroci da invertire per sciogliere  $K$



# Chirurgia e polinomi



$$V(\text{nodo banale}) = 1$$

$$t^{-1} V(K_+) - t V(K_-) = (t^{-1/2} - t^{1/2}) V(K_0)$$

Diagram illustrating a knot  $K$  with a crossing, and its resolutions  $K_+$  and  $K_-$ . The knot  $K$  is shown on the left, with a crossing marked with a plus sign. The resolutions  $K_+$  and  $K_-$  are shown on the right, with a plus sign and a minus sign respectively. The knot  $K$  is shown to be non-trivial.

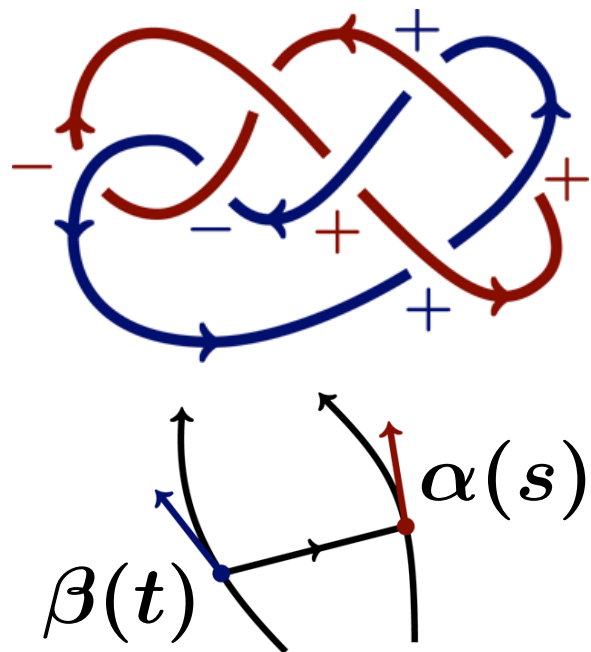
$$V(K) = -t^4 + t^3 + t$$

$\Downarrow$

$K$  non è banale



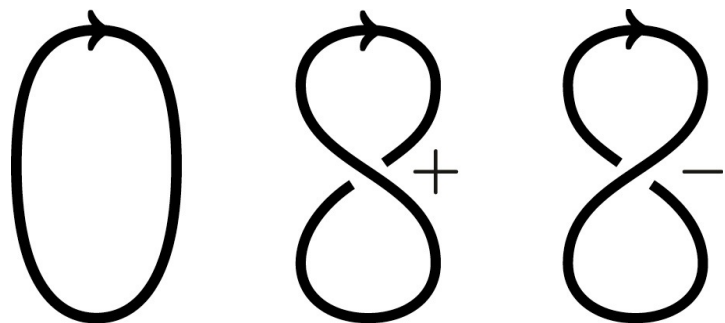
# Indice di allacciamento



$$\text{Lk}(K_1, K_2) = \left| \left\{ \begin{array}{c} \text{red} \\ \text{blue} \end{array} \right\} \right| - \left| \left\{ \begin{array}{c} \text{blue} \\ \text{red} \end{array} \right\} \right|$$

$$= \frac{1}{4\pi} \int \frac{\alpha(s) - \beta(t)}{\|\alpha(s) - \beta(t)\|^3} \cdot \alpha'(s) \times \beta'(t) ds dt$$

$\text{Lk}(K_1, K_2) = \text{Lk}(K_2, K_1)$  è un invariante topologico



$$\text{Wr}(K) = \left| \left\{ \begin{array}{c} \text{red} \\ \text{red} \end{array} \right\} \right| - \left| \left\{ \begin{array}{c} \text{blue} \\ \text{blue} \end{array} \right\} \right|$$

$\text{Wr}(K)$  dipende dal diagramma (dalla proiezione)



# Nodi geometrici

*Curve chiuse nello spazio a meno di movimenti rigidi*



$$C(K) = \frac{1}{4\pi} \int_{S^2} \left( \left| \left\{ \begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \end{array} \right\} \right| + \left| \left\{ \begin{array}{c} \text{diagram 3} \\ \text{diagram 4} \end{array} \right\} \right| \right) d\sigma$$

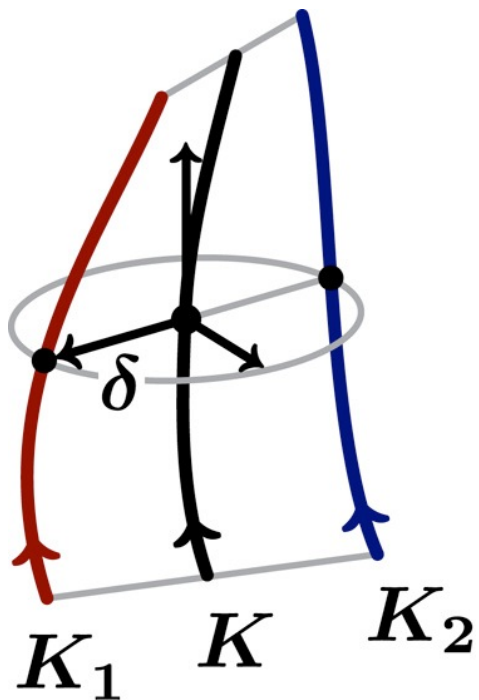
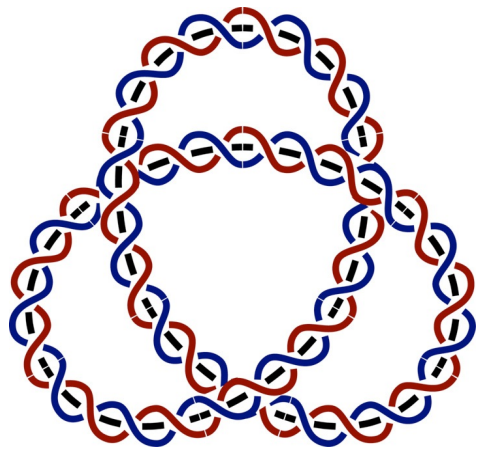
$$Wr(K) = \frac{1}{4\pi} \int_{S^2} \left( \left| \left\{ \begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \end{array} \right\} \right| - \left| \left\{ \begin{array}{c} \text{diagram 3} \\ \text{diagram 4} \end{array} \right\} \right| \right) d\sigma$$

$$= \frac{1}{4\pi} \int \frac{\alpha(s) - \alpha(t)}{\|\alpha(s) - \alpha(t)\|^3} \cdot \alpha'(s) \times \alpha'(t) ds dt$$

I numeri medi di incroci (con segno)  
 $C(K)$  e  $Wr(K)$  sono invarianti geometrici



# Teorema di White



$$\text{Tw}(K) = \frac{1}{2\pi} \int \frac{\delta'(s) \times \delta(s)}{\|\delta(s)\|^2} \cdot \alpha'(s) ds$$

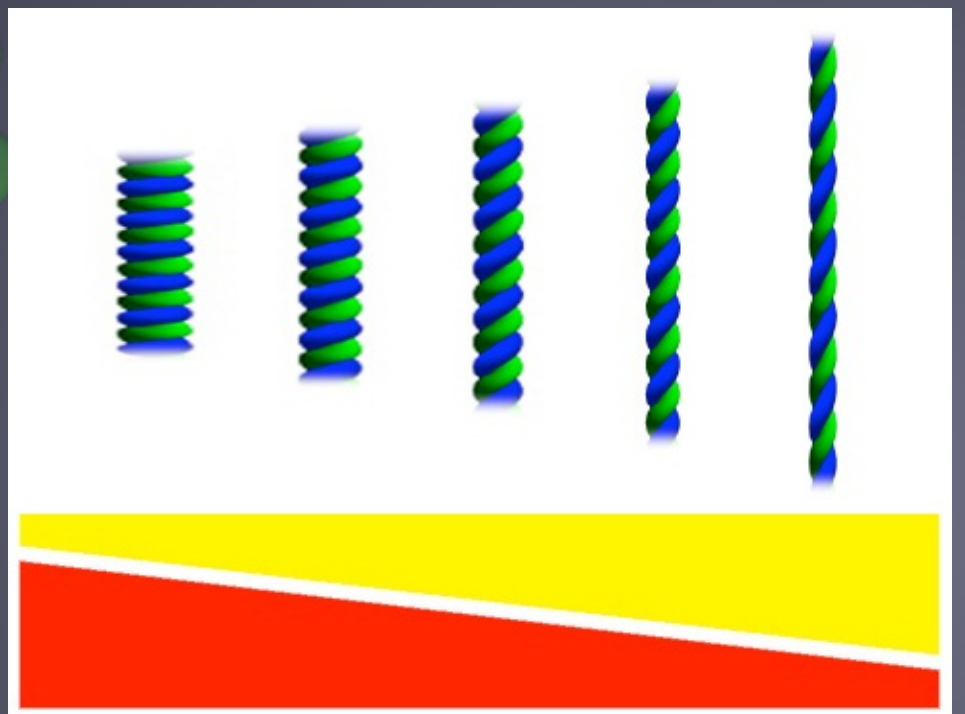
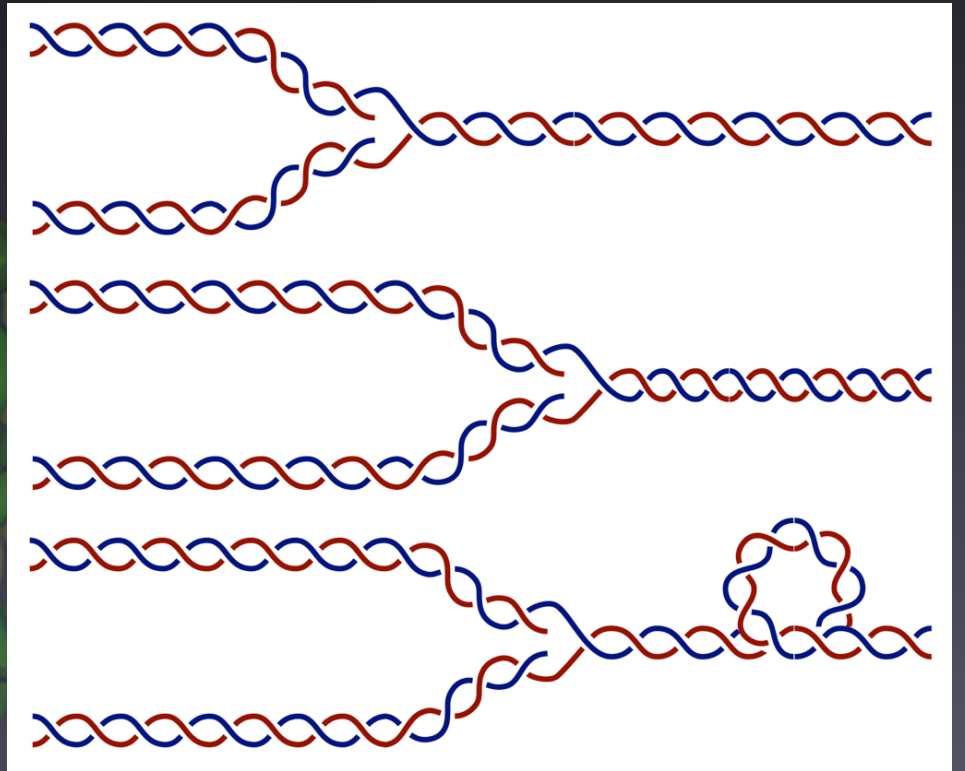
L'indice di avvitamento  $\text{Tw}(K)$   
è un invariante geometrico  
che conta i giri della doppia elica

$$\text{Lk}(K) = \text{Lk}(K, K_{1,2}) = \text{Lk}(K_1, K_2)$$

$$\text{Wr}(K) + \text{Tw}(K) = \text{Lk}(K)$$



# Super-avvitamento

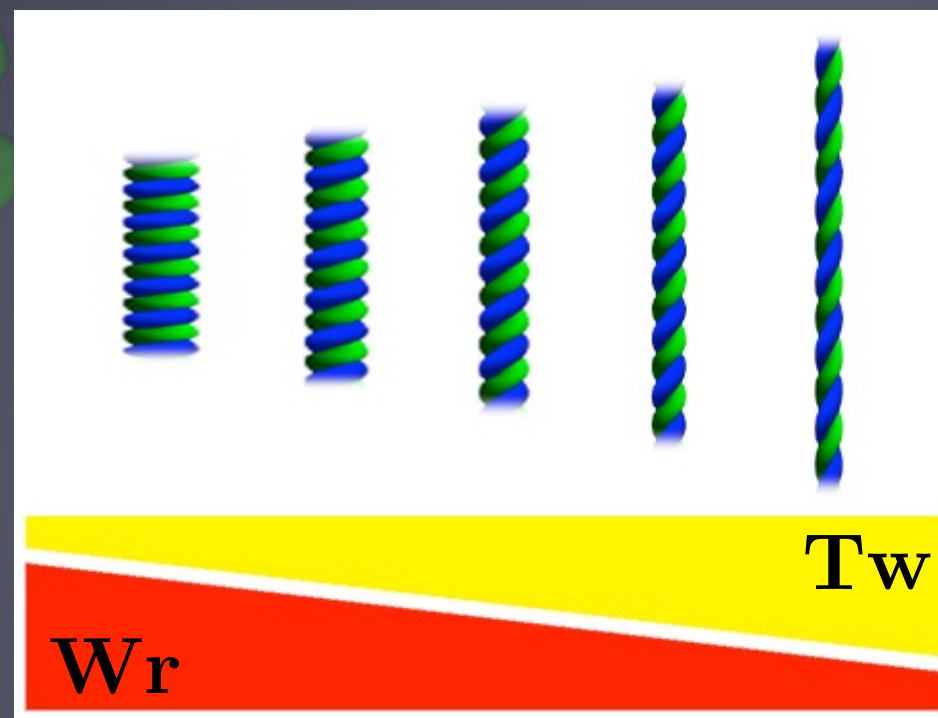
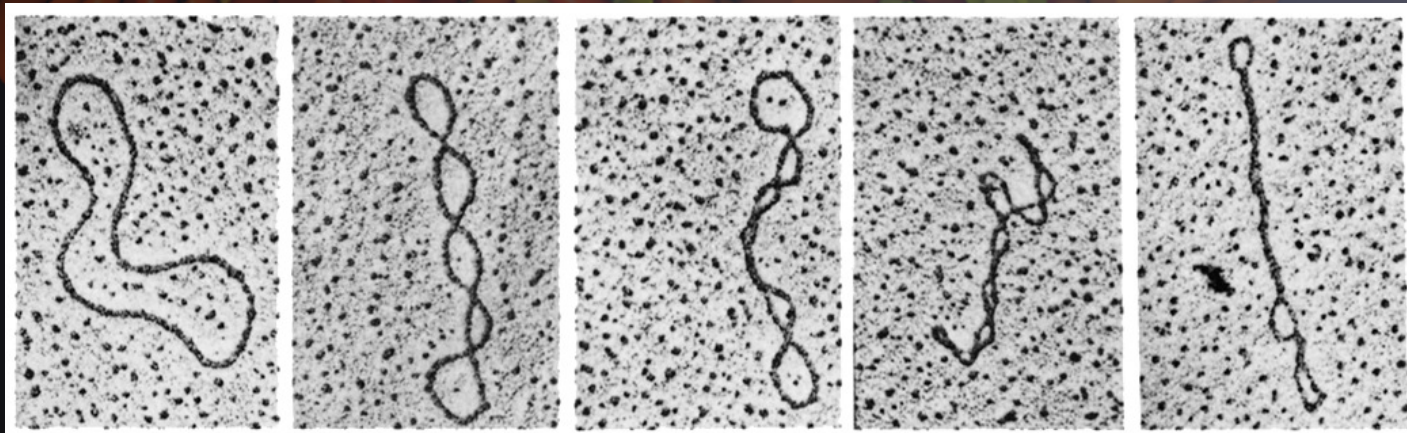
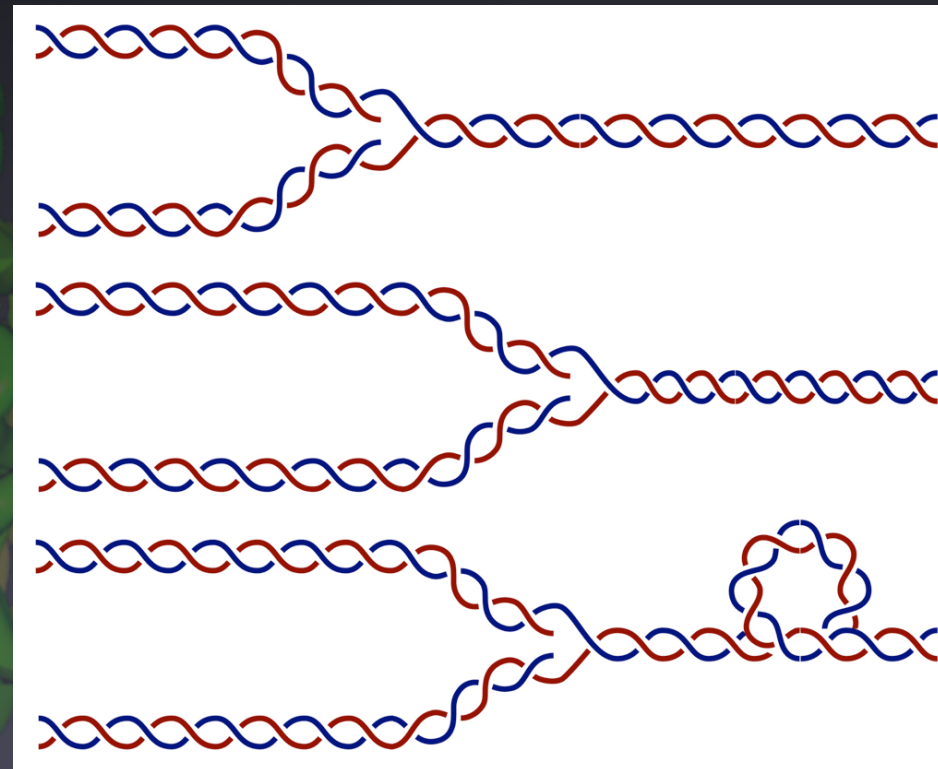
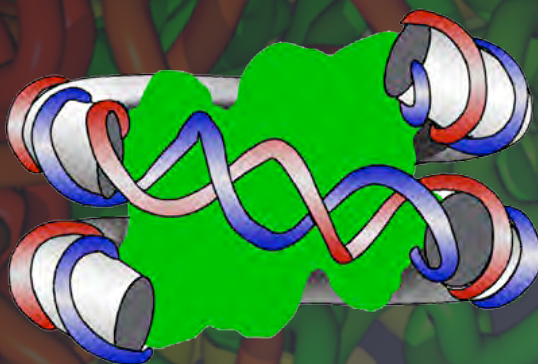
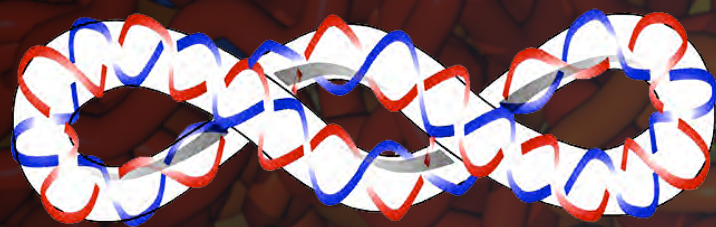




# Super-avvitamento

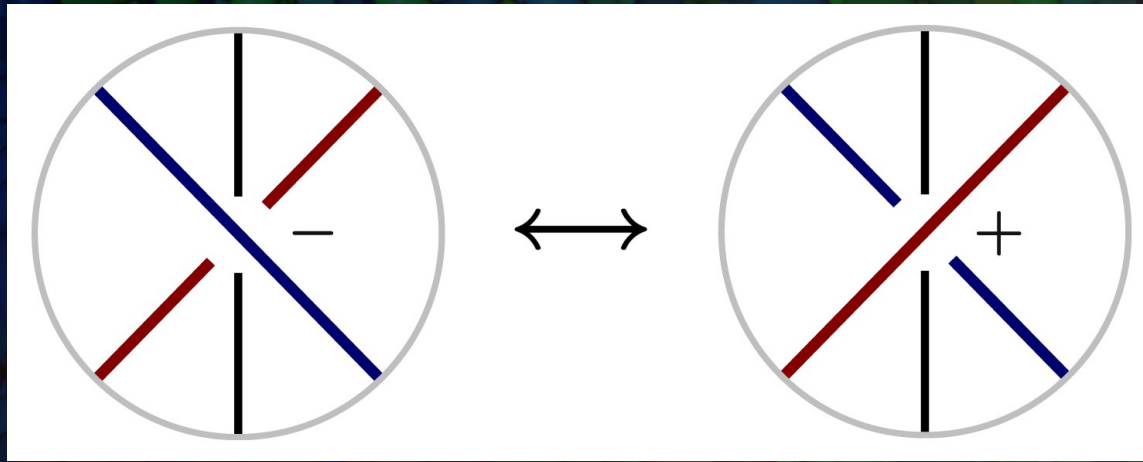
$$\sigma(K) = \frac{\text{Lk}(K) - \text{Lk}(K_0)}{\text{Lk}(K_0)}$$

$$\sigma(K) \sim -6\% \left( \frac{3}{4} W_r \text{ e } \frac{1}{4} T_w \right)$$





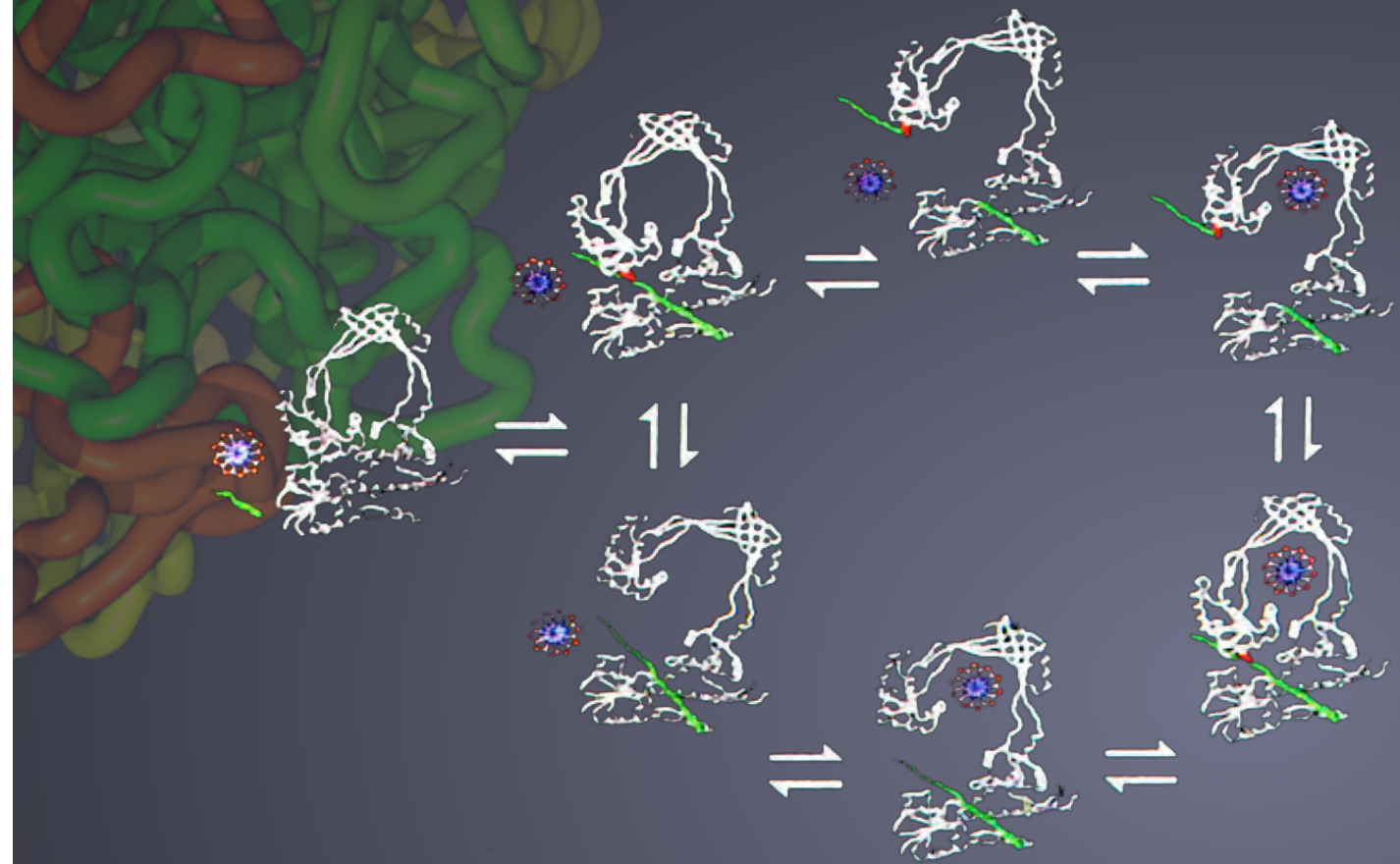
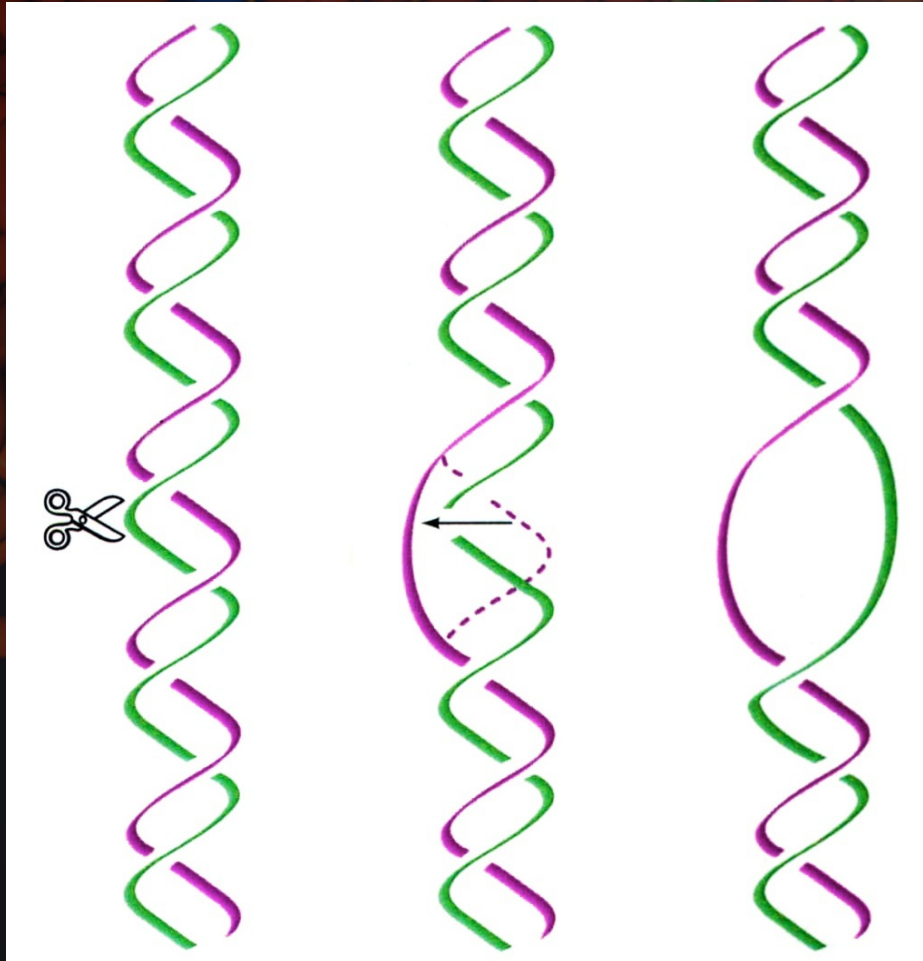
# Topoisomerasi I



$$\Delta Lk = \pm 1$$

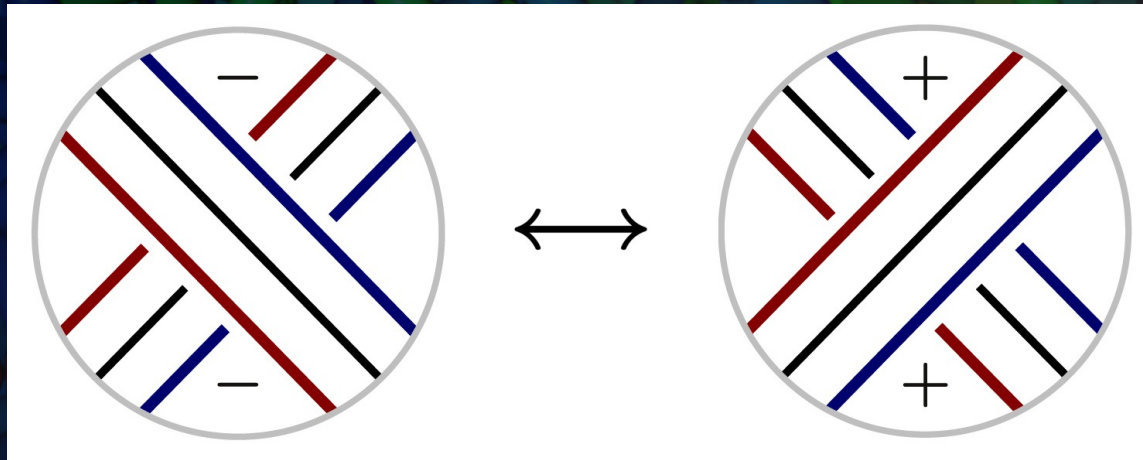
$$\Delta Wr \sim 0$$

$$\Delta Tw \sim \pm 1$$





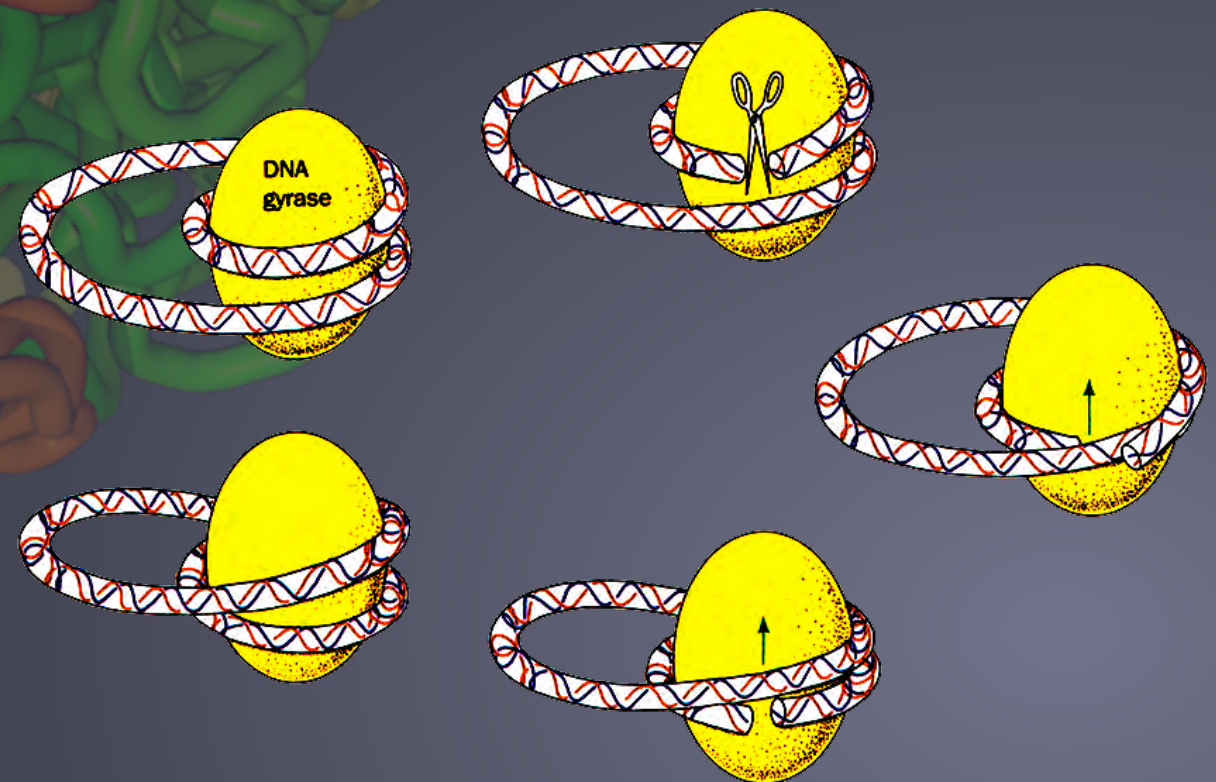
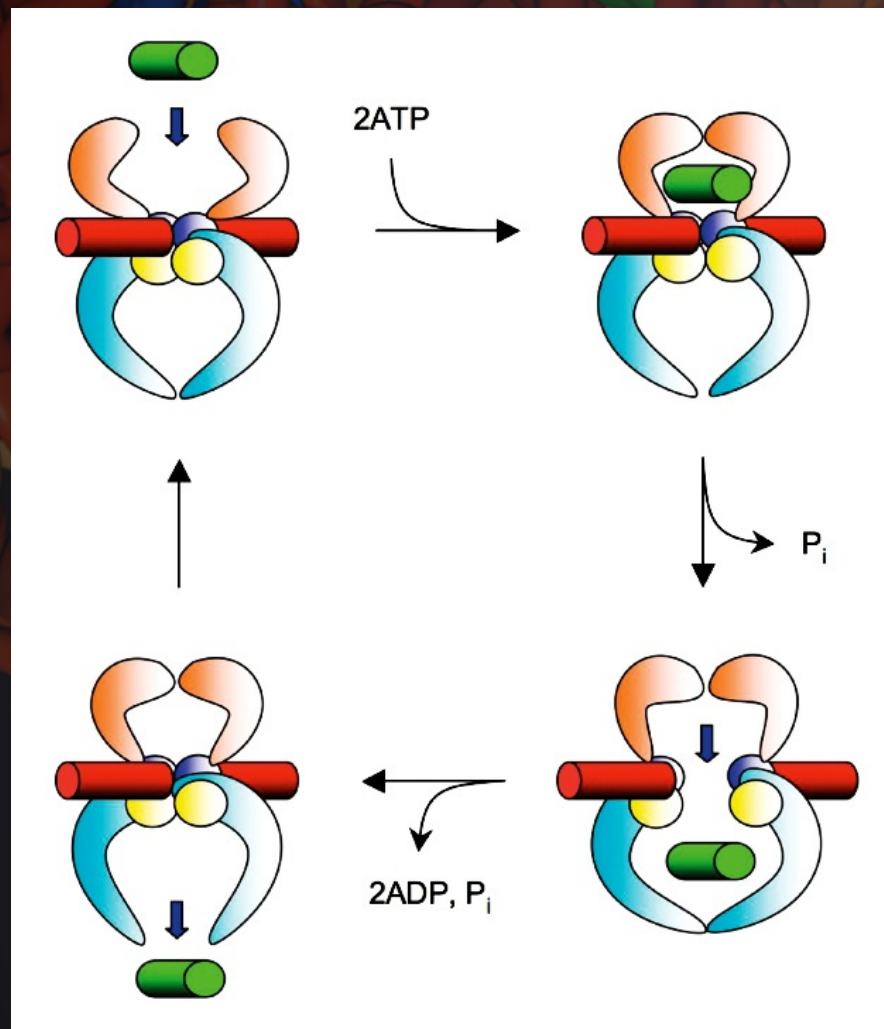
# Topoisomerasi II



$$\Delta Lk = \pm 2$$

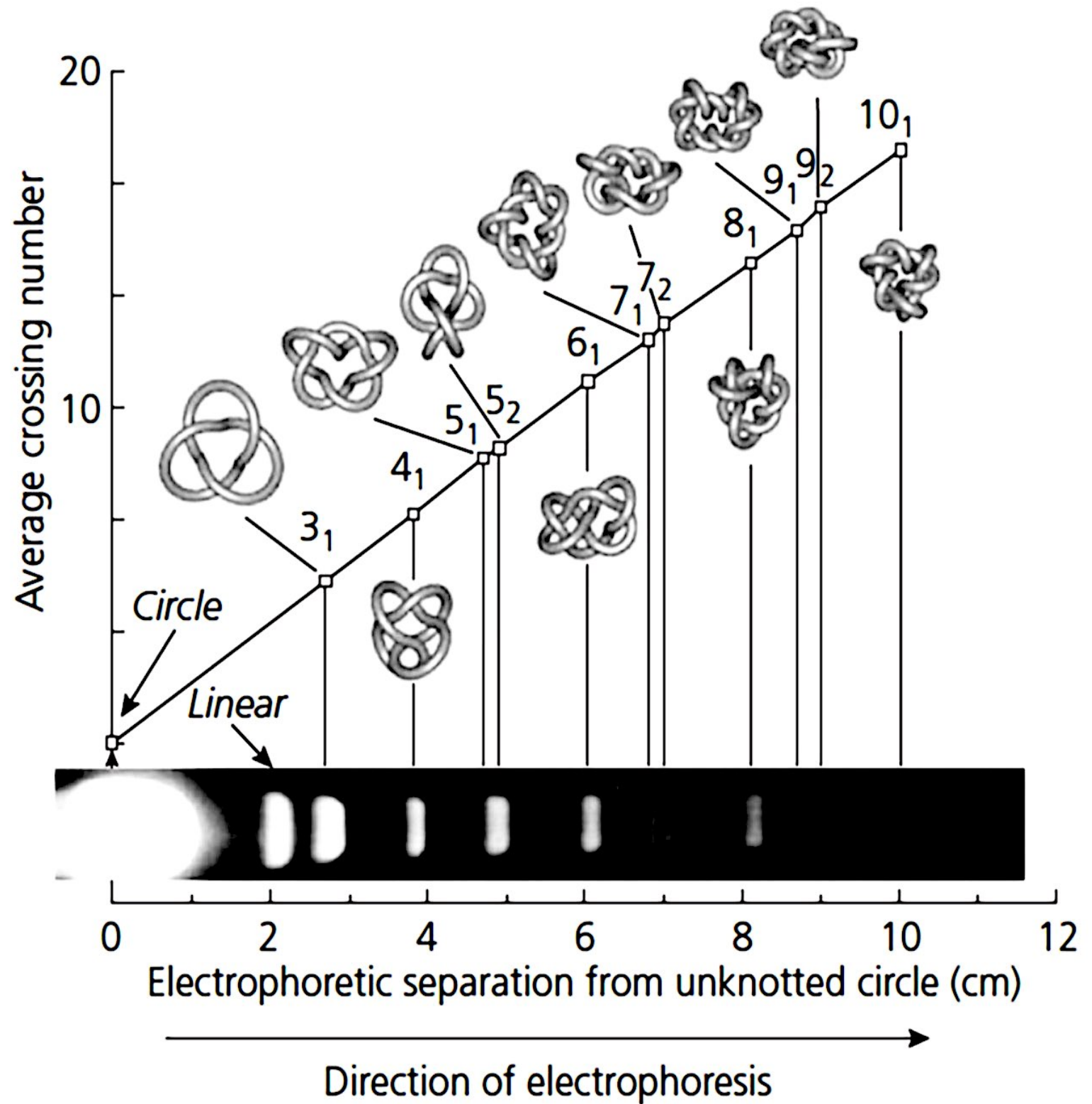
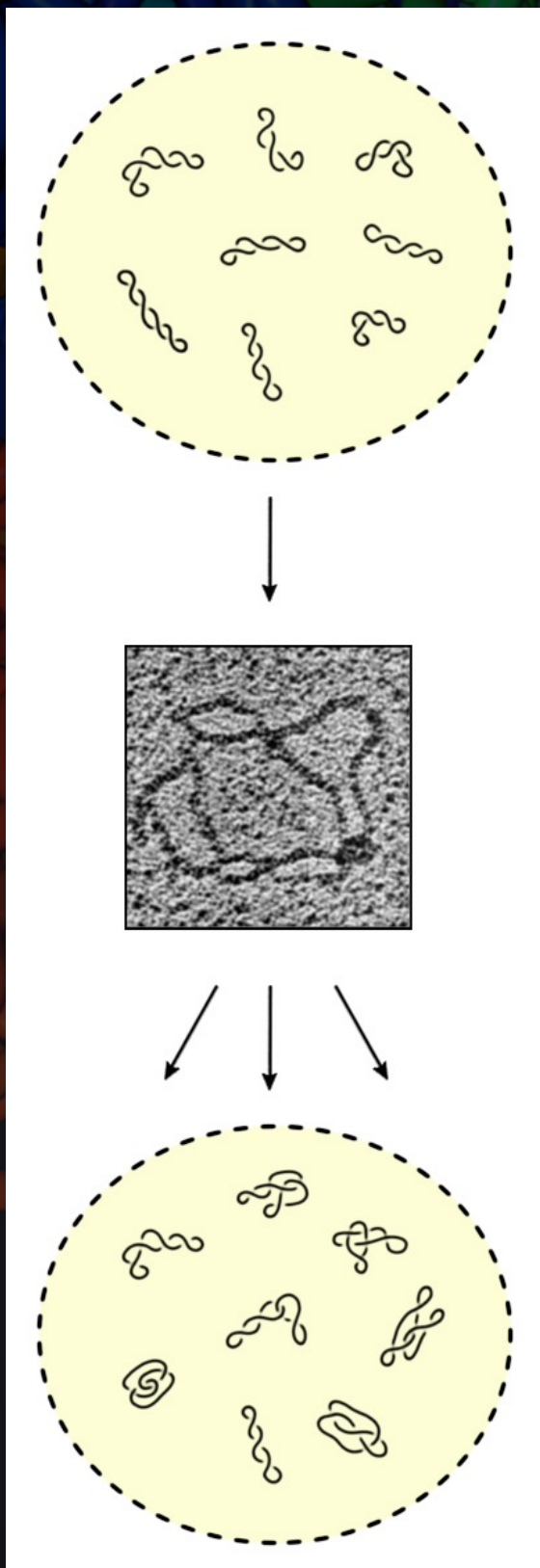
$$\Delta Wr \sim \pm 2$$

$$\Delta Tw \sim 0$$





# Ricombinasi e nodi









# Matasse razionali



$T(3)$



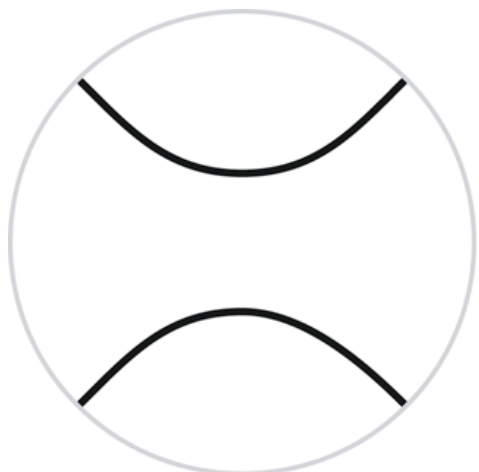
$T(2, 3)$



$T(-4, 2, 3)$



$T(-1, -4, 2, 3)$



$T(0)$



$T(0, 0)$



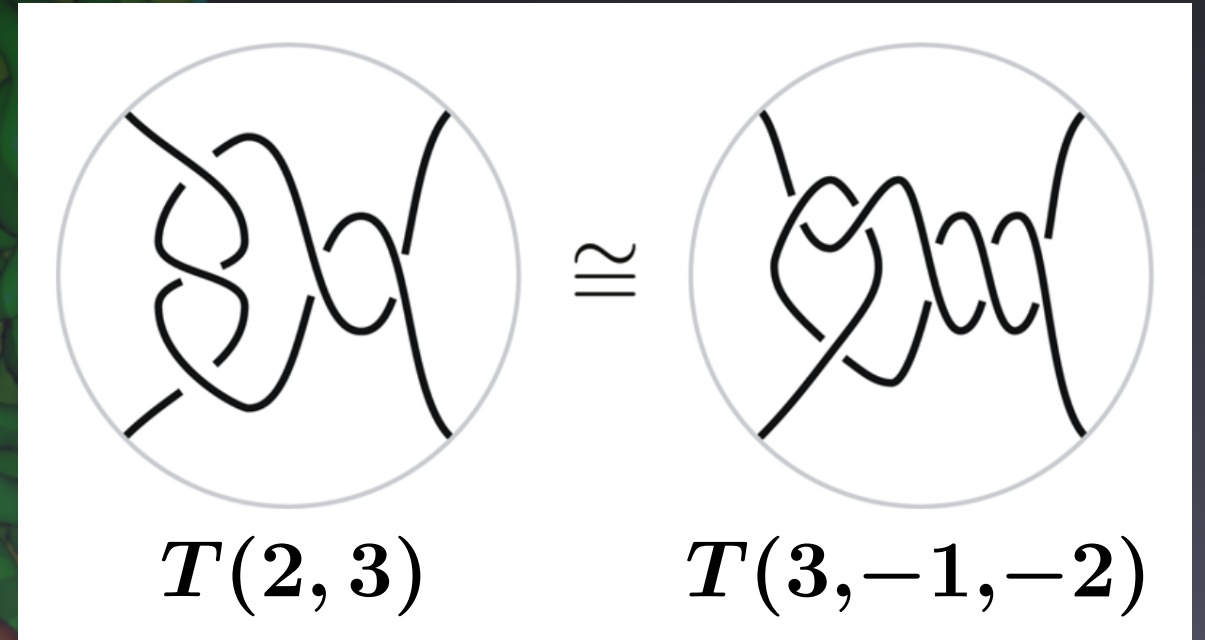
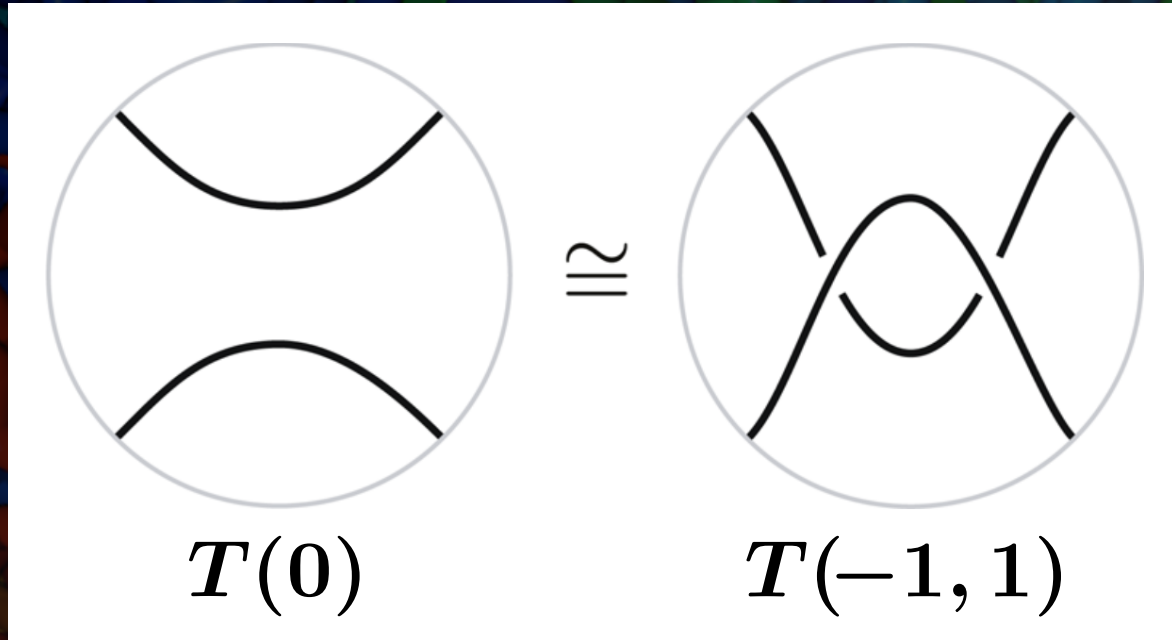
$T(0, 3)$



irrazionale



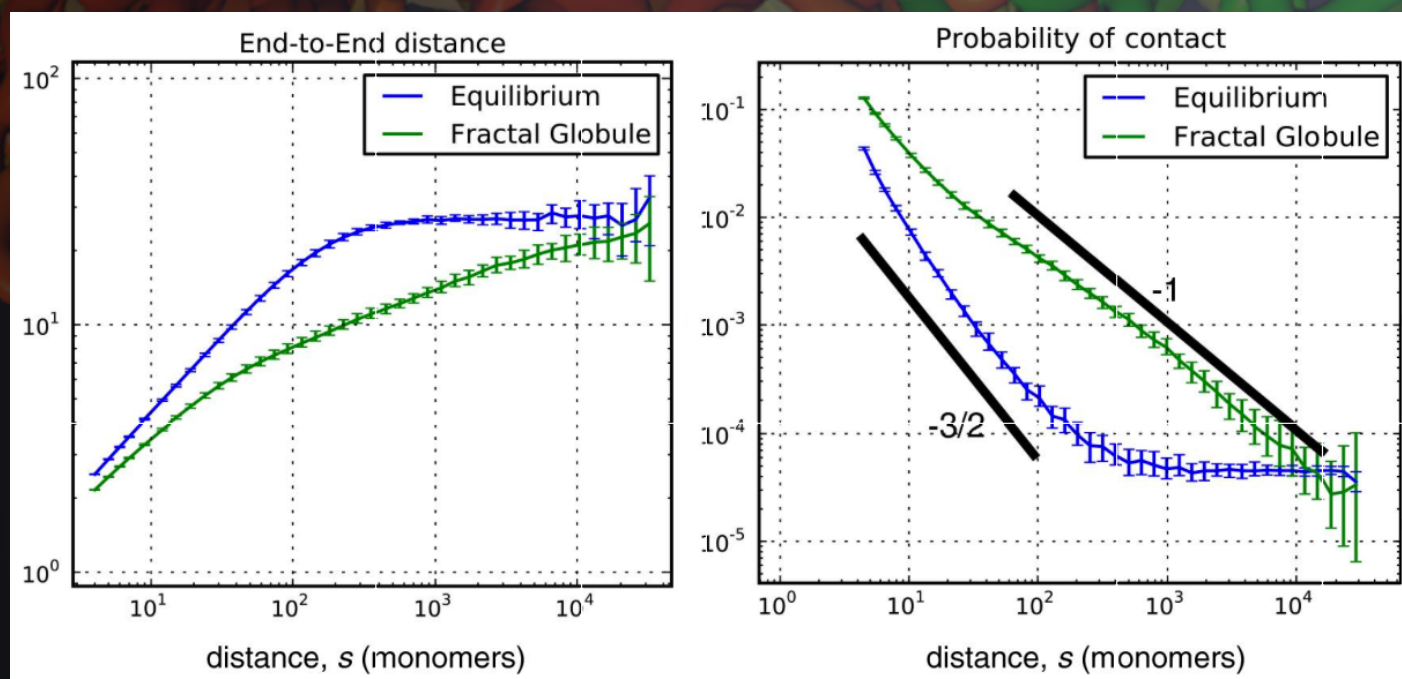
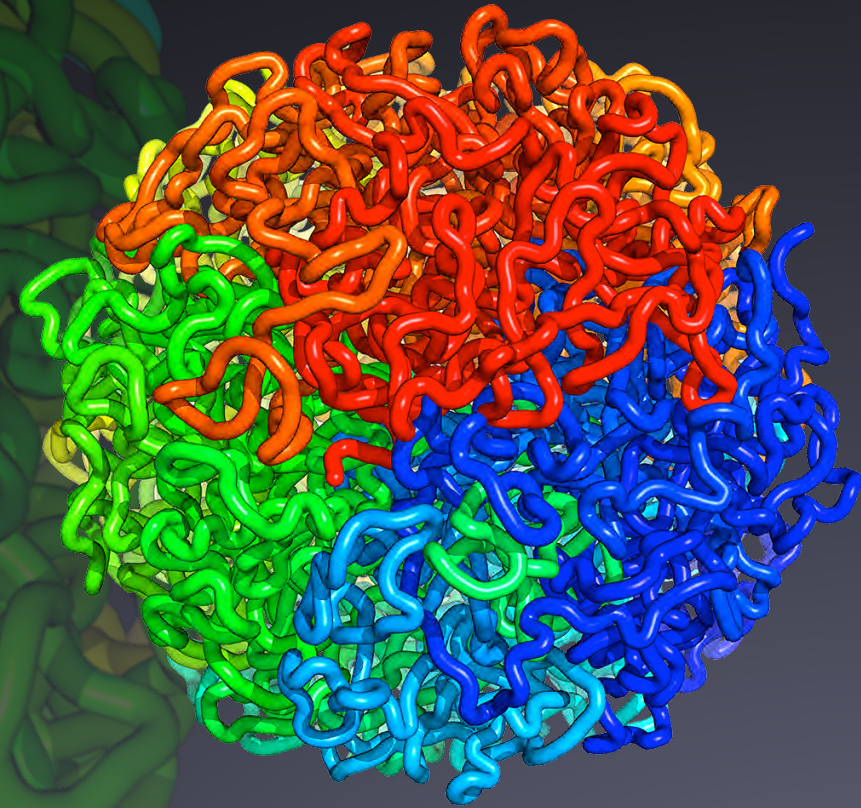
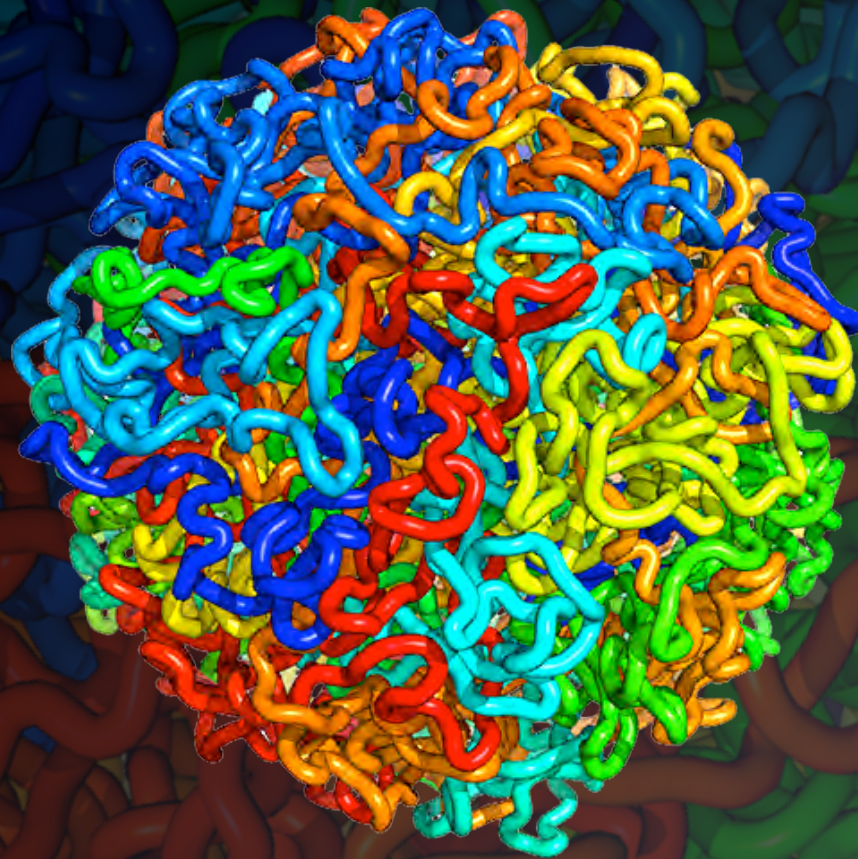
# Equazioni topologiche



$$T(a_1, a_2, \dots, a_n) \iff a_1 + \frac{1}{a_2 + \frac{1}{\dots + \frac{1}{a_{n-1} + \frac{1}{a_n}}}}$$



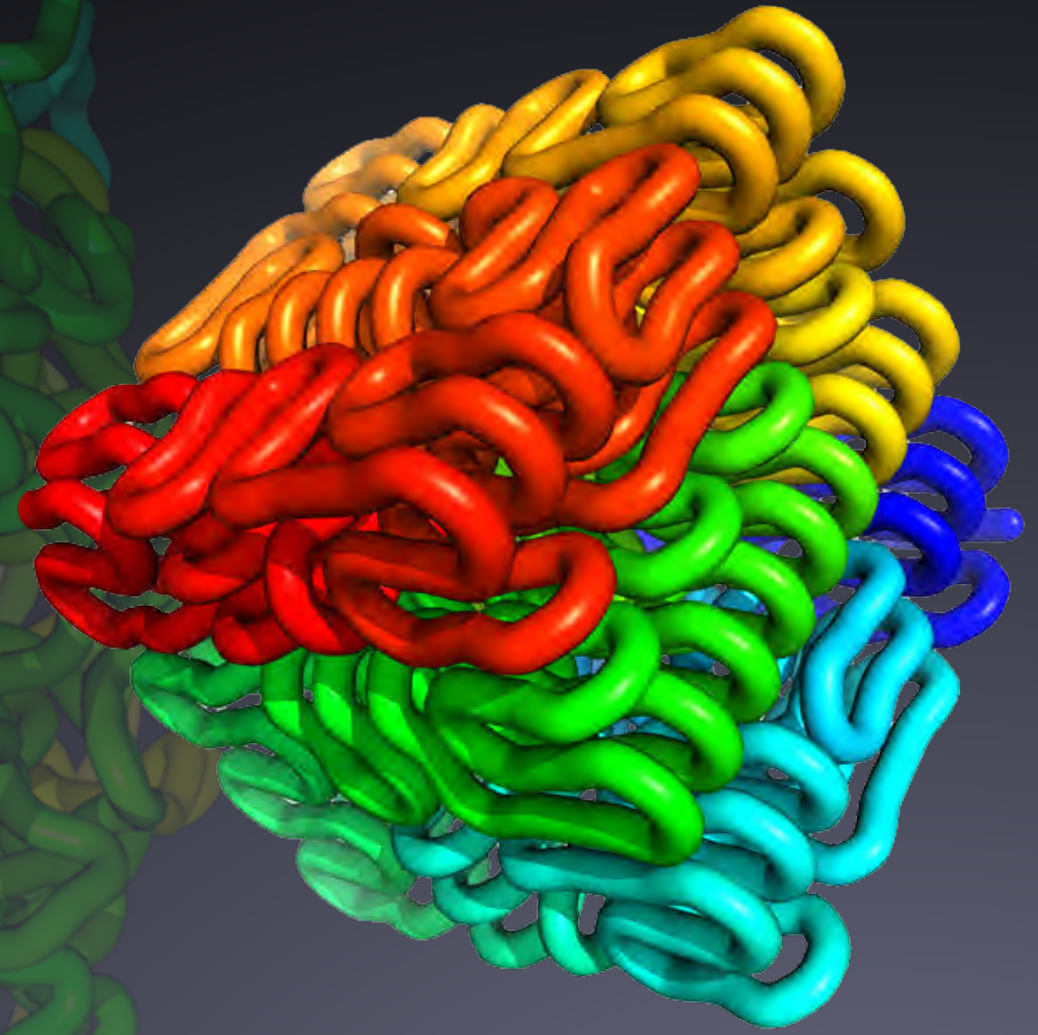
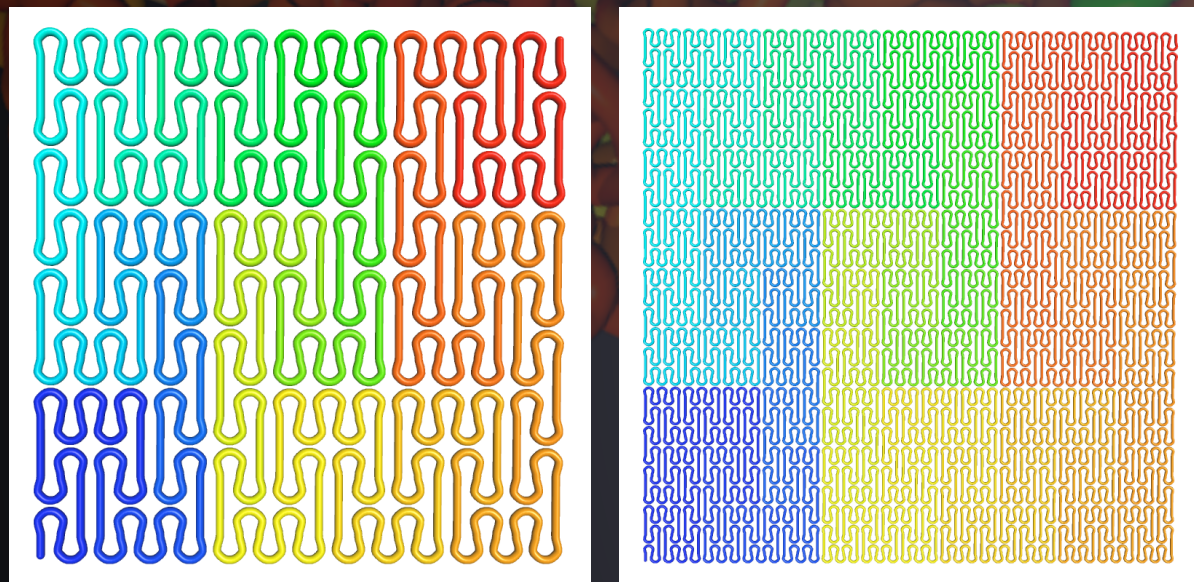
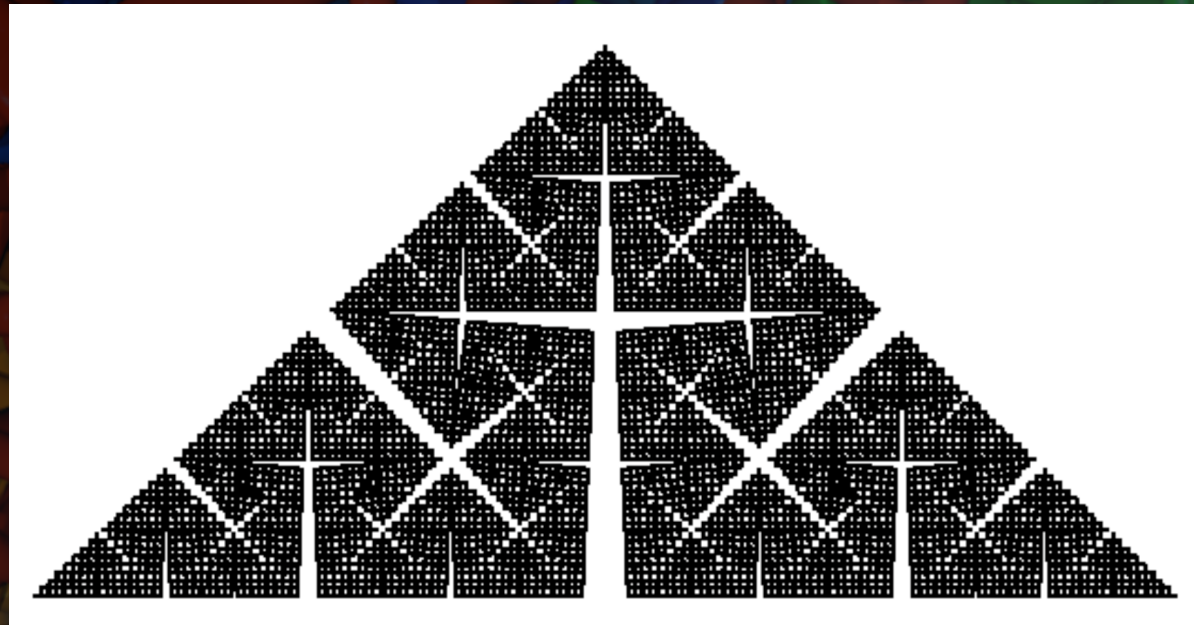
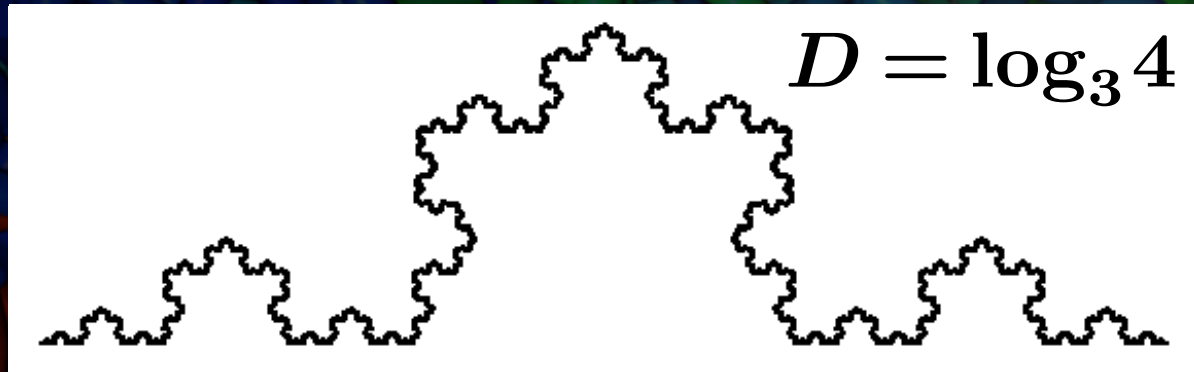
# Organizzazione del genoma



**Compartimentato**  
**Supercompatto**  
**Accessibile**  
**Non annodato**



# Struttura frattale



DNA compatto

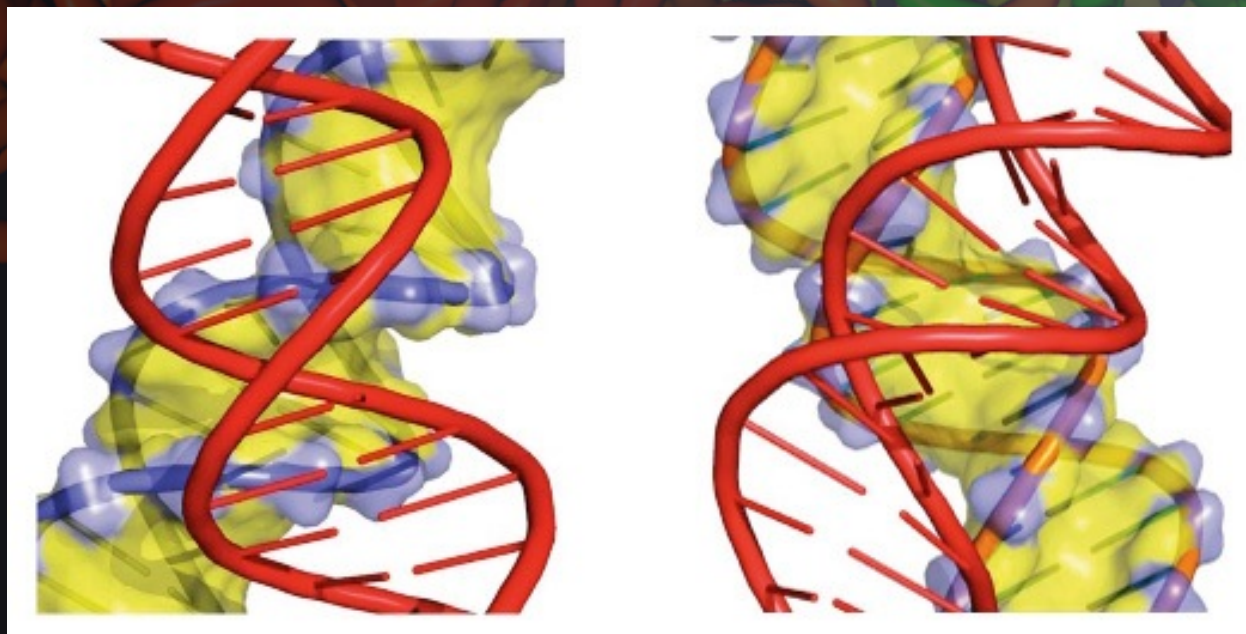
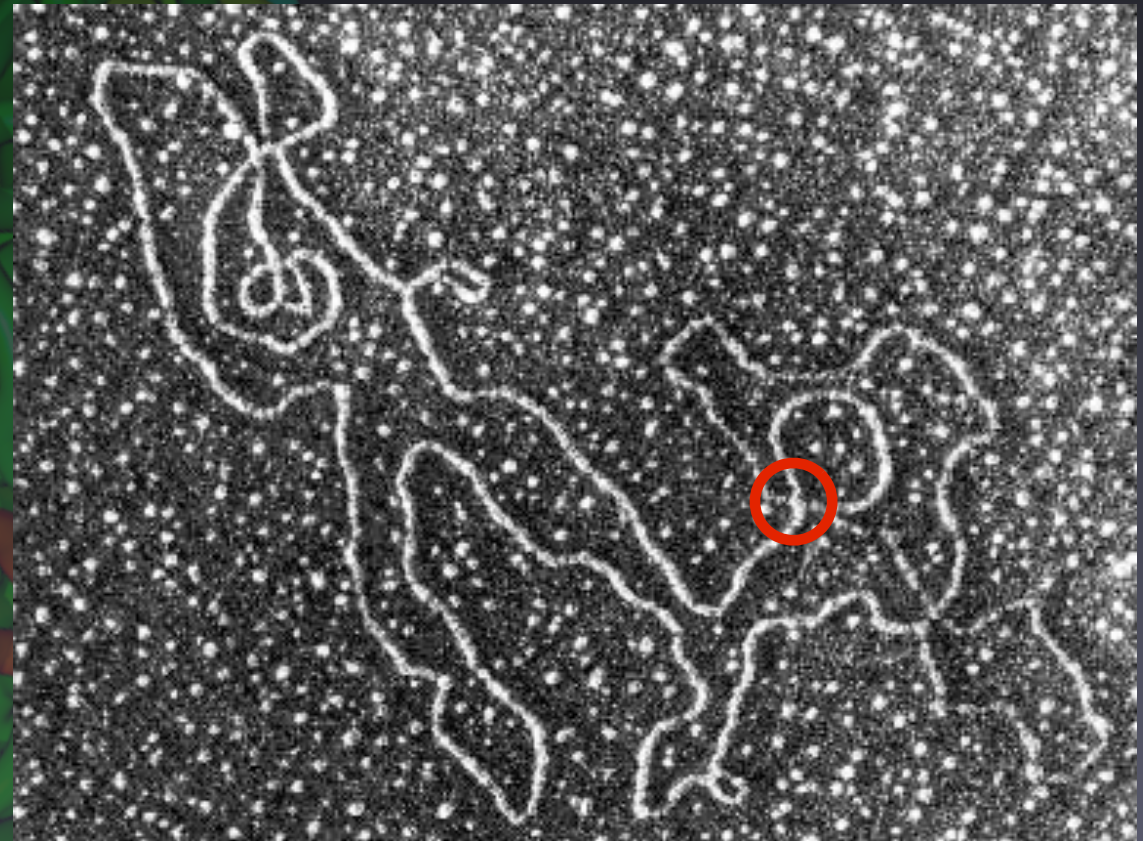
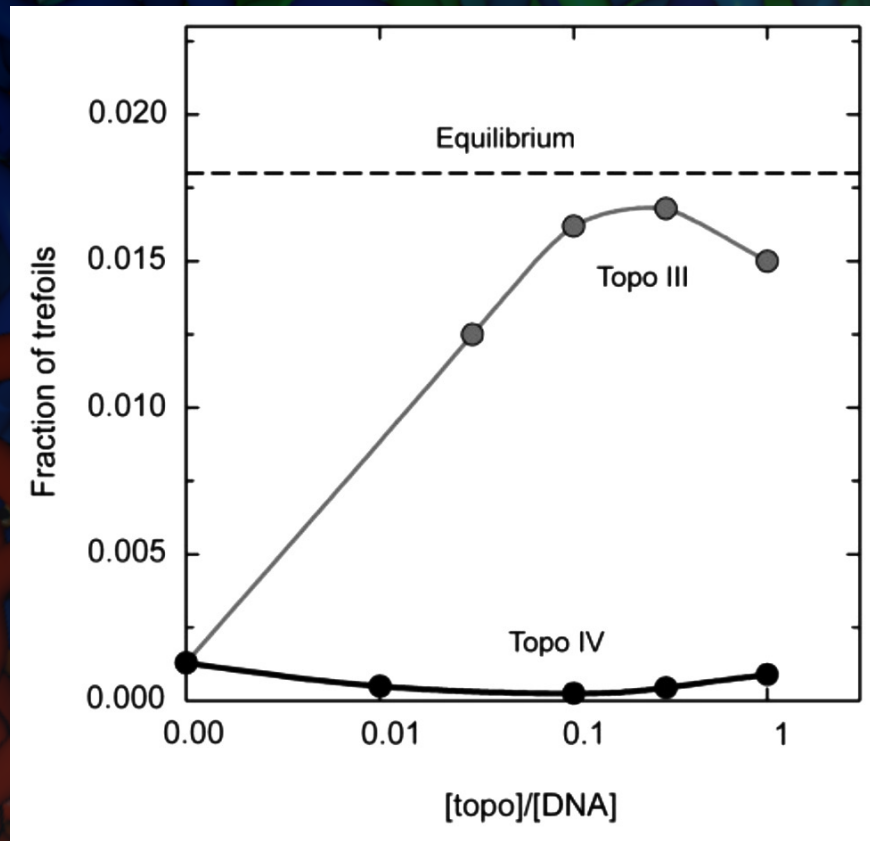
$$2 < D < 3$$

DNA scompartato

$$1 < D < 2$$



# Semplificazione topologica





# Vita nello spazio



## *Teorema (Gromov)*

In uno stagno “generico”  
contenente  $1000 \text{ m}^3$  di acqua,  
le sostanze chimiche necessarie  
alla vita e sufficiente energia libera,  
la probabilità che non vi siano forme  
di vita unicellulare è minore di  $10^{-100}$

