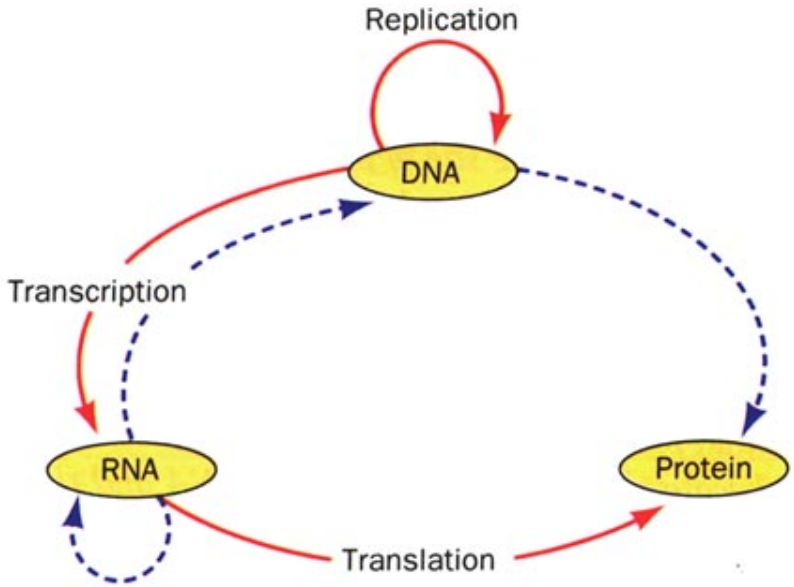
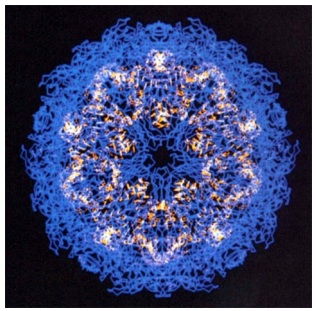
La Matematica nella cultura del 2000

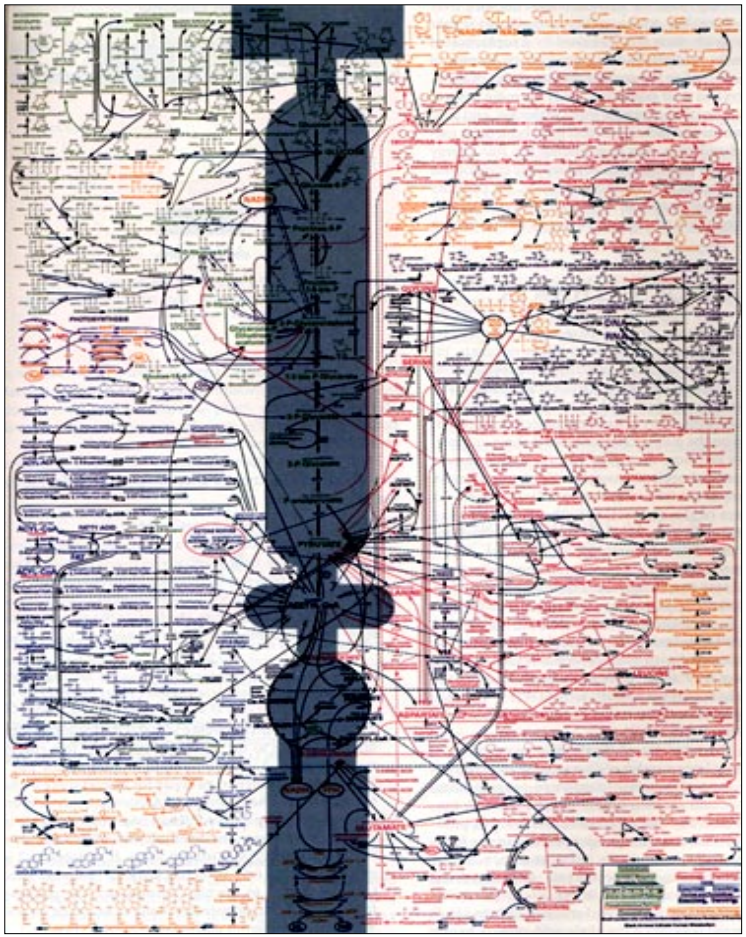
MATEMATICA E BIOLOGIA

La geometria del DNA

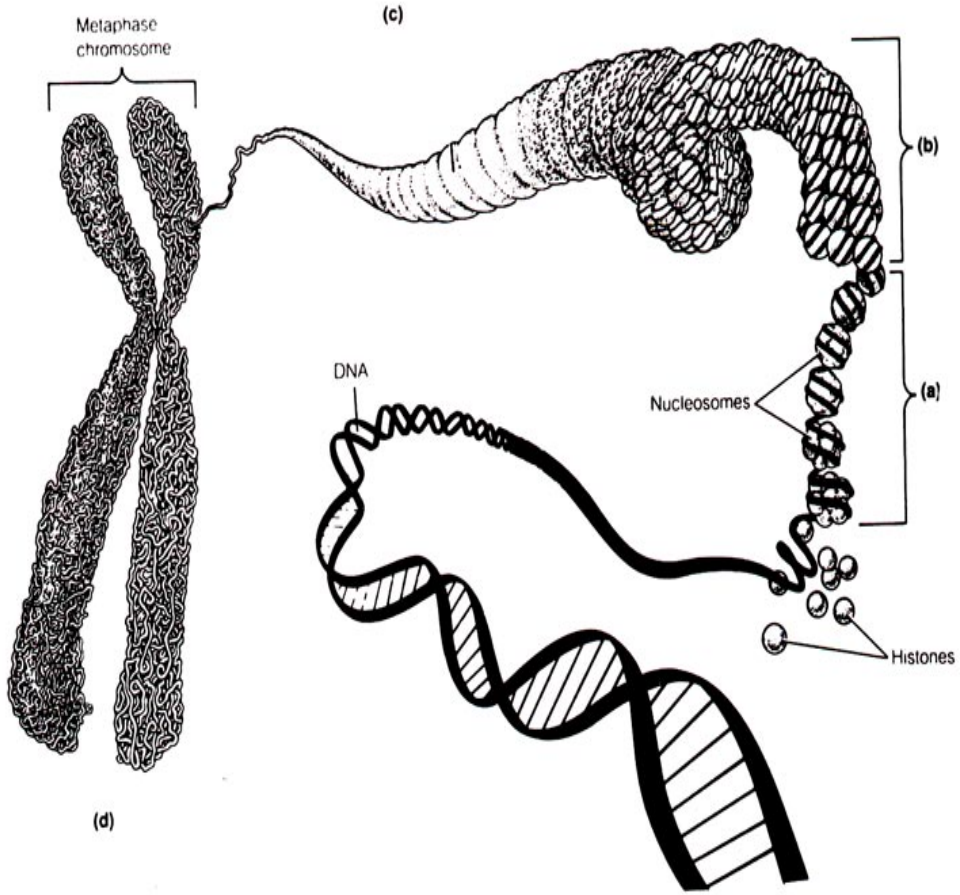
Il dogma centrale







I cromosomi

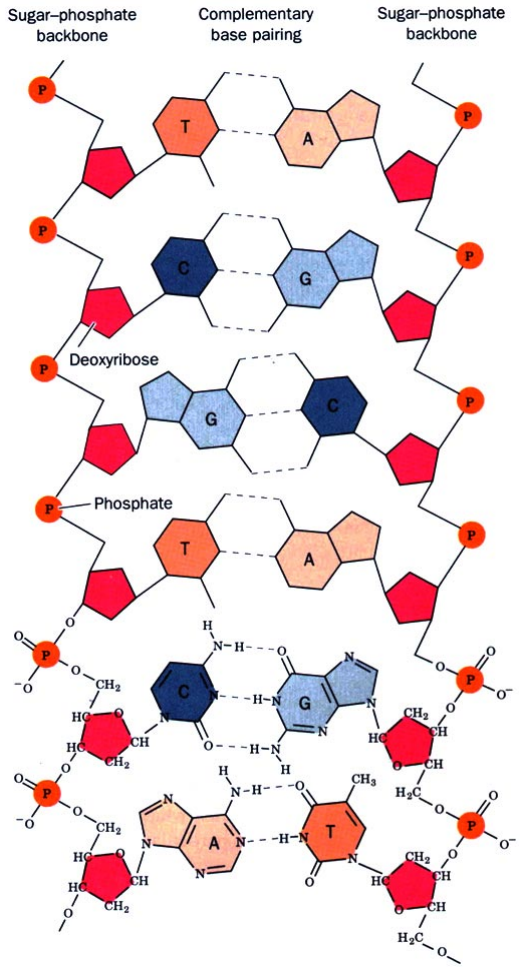


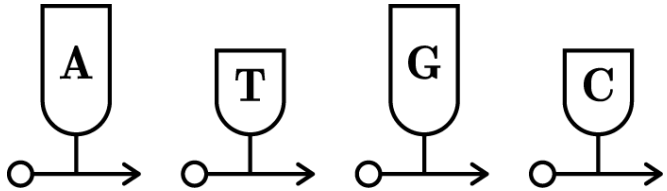
4/5 livelli con fattore di scala $\sim 1/10$ ognuno

lung. $\sim 1m$

$$\mathrm{Dim_T} = 1$$
$$\mathrm{Dim_H} > 2$$

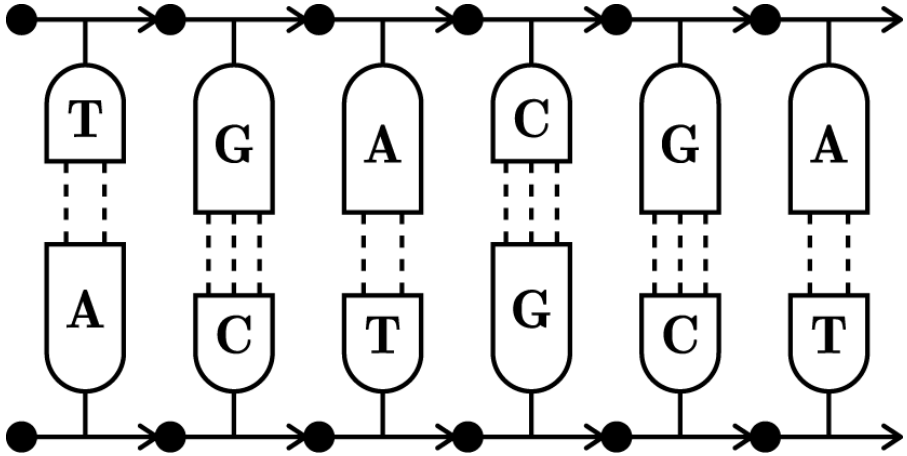
DNA e codice genetico



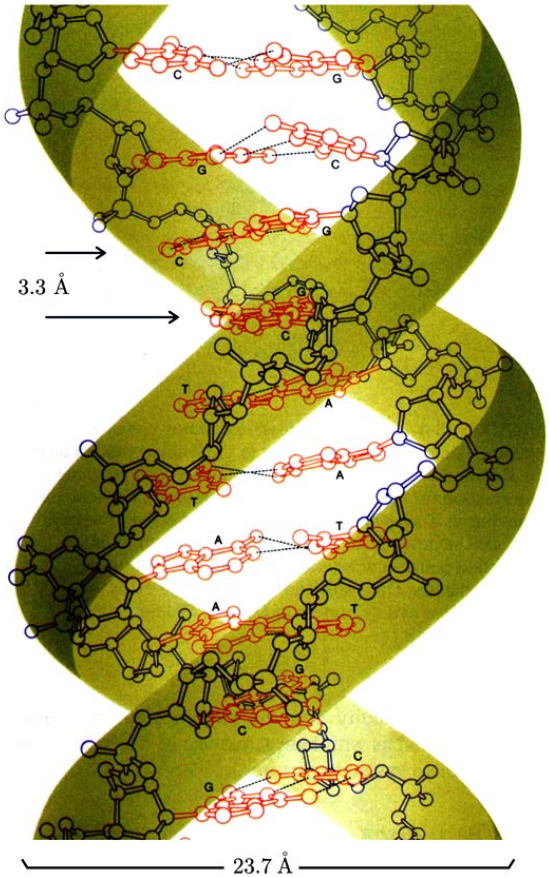


ADENINA ⇝ TIMINA

GUANINA ⇝ CITOSINA

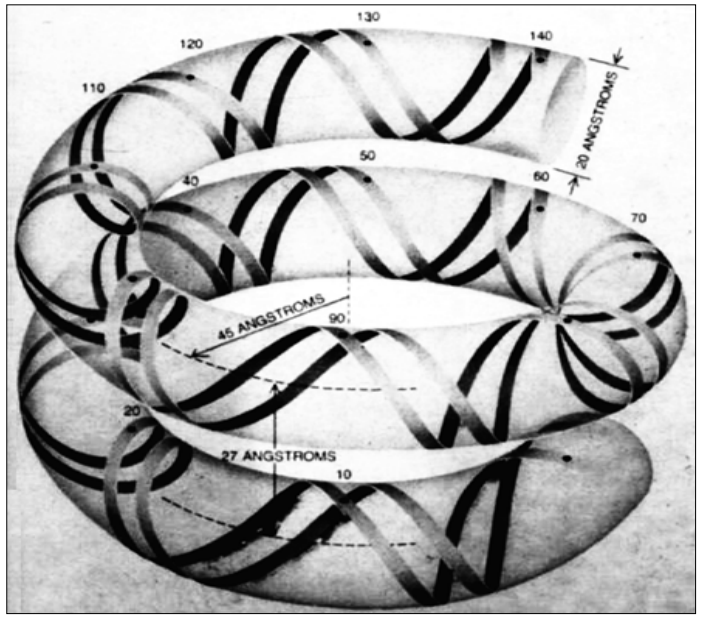


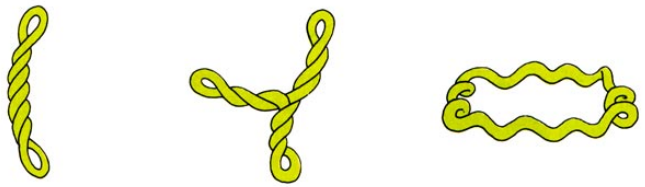
La doppia elica



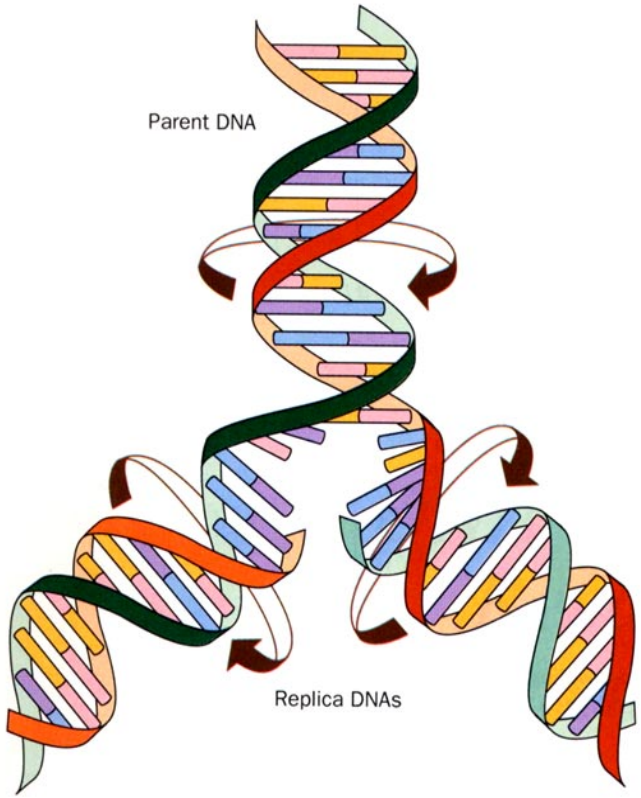
$p = 10$ basi (10.5 in sol.)

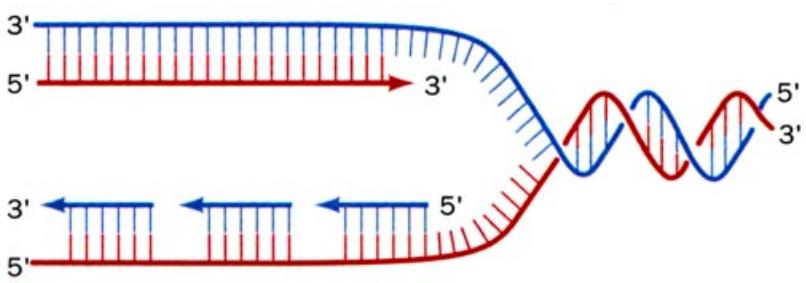
Super-avvitamento





La duplicazione





100-1000 basi/sec

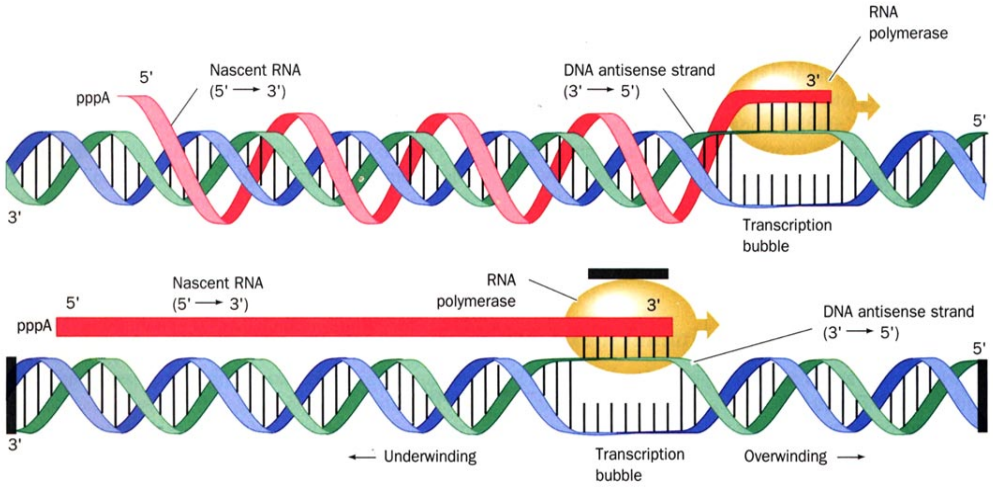
Computer molecolare

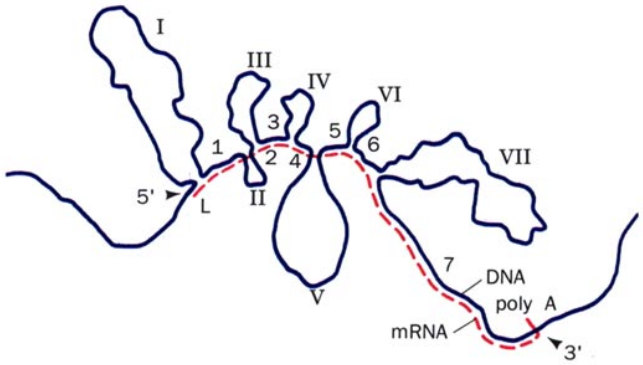
10^{20} oper/sec

10^{24} byte/cm^2

La trascrizione dei geni

Gene = segmento di DNA $\rightsquigarrow$ RNA





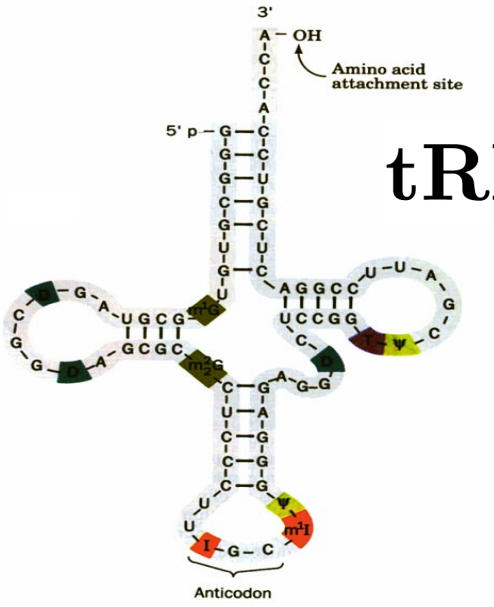
Esoni = sequenze espresse

Introni = sequenze inspresse

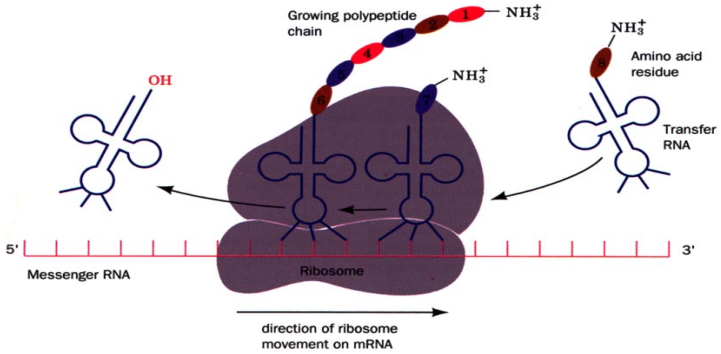
Traduzione e sintesi proteica

60 codoni $\rightsquigarrow$ 20 aminoacidi

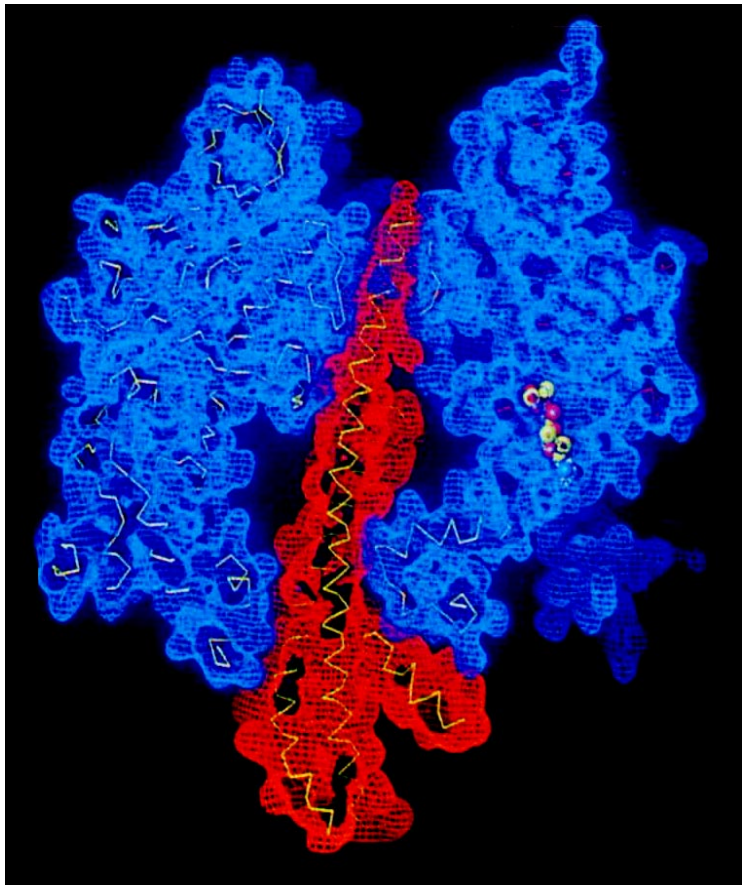
First position (5' end)	Second position				Third position (3' end)
	U	**C**	**A**	**G**	
U	UUU Phe	UCU Ser	UAU Tyr	UGU Cys	U
	UUC Phe	UCC Ser	UAC Tyr	UGC Cys	C
	UUA Leu	UCA Ser	UAA Stop	UGA Stop	A
	UUG Leu	UCG Ser	UAG Stop	UGG Trp	G
C	CUU Leu	CCU Pro	CAU His	CGU Arg	U
	CUC Leu	CCC Pro	CAC His	CGC Arg	C
	CUA Leu	CCA Pro	CAA Gln	CGA Arg	A
	CUG Leu	CCG Pro	CAG Gln	CGG Arg	G
A	AUU Ile	ACU Thr	AAU Asn	AGU Ser	U
	AUC Ile	ACC Thr	AAC Asn	AGC Ser	C
	AUA Ile	ACA Thr	AAA Lys	AGA Arg	A
	AUG Met[b]	ACG Thr	AAG Lys	AGG Arg	G
G	GUU Val	GCU Ala	GAU Asp	GGU Gly	U
	GUC Val	GCC Ala	GAC Asp	GGC Gly	C
	GUA Val	GCA Ala	GAA Glu	GGA Gly	A
	GUG Val	GCG Ala	GAG Glu	GGG Gly	G



tRNA

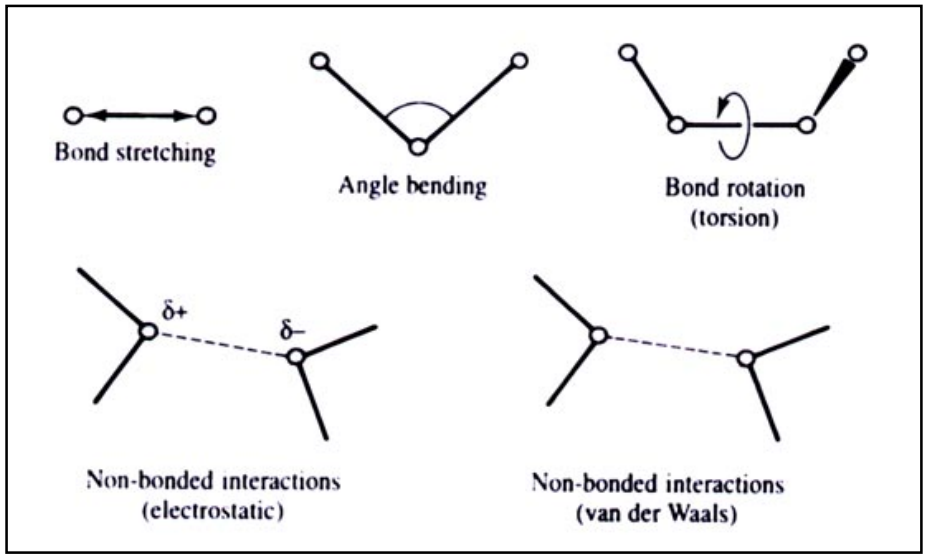


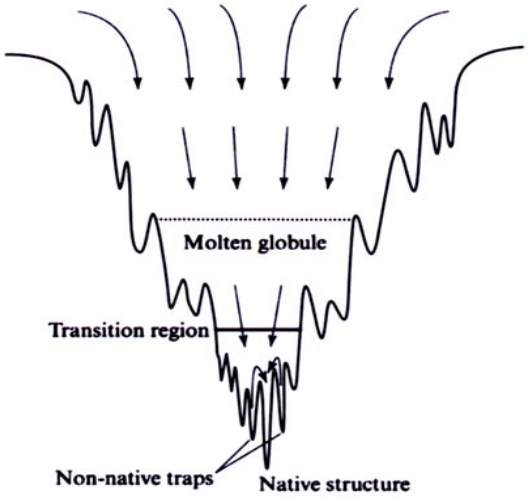
La struttura delle proteine



Modelli molecolari

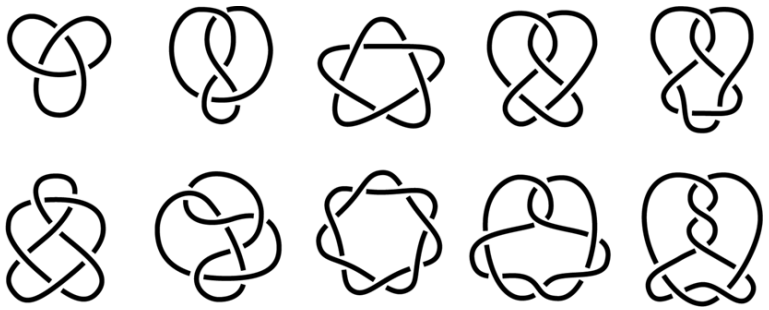
$$E = \sum_{leg} C_l(r_l - r_l^0)^2 + \sum_{ang} C_a(\theta_a - \theta_a^0)^2 + \sum_{tor} C_t \cos(n_t\omega_t - \phi_t) +$$
$$+ \sum_{i<j} \frac{q_i q_j}{D r_{ij}} + \sum_{i<j} 4D_{ij}\left(\left(\frac{C_{ij}}{r_{ij}}\right)^{12} - \left(\frac{C_{ij}}{r_{ij}}\right)^{6}\right)$$

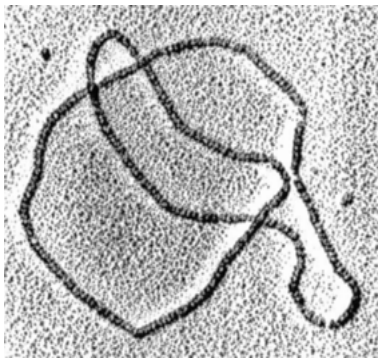


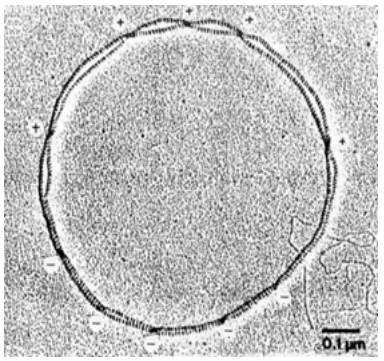


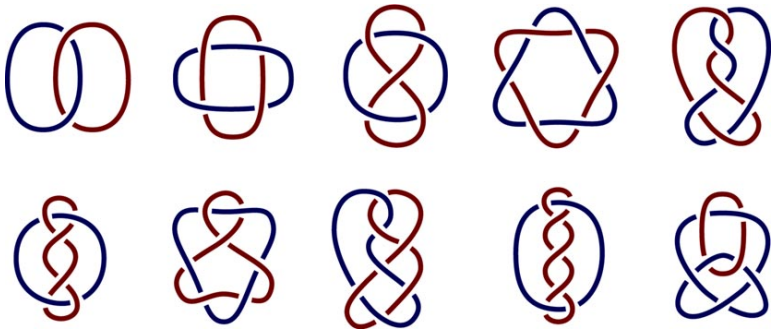
Nodi topologici

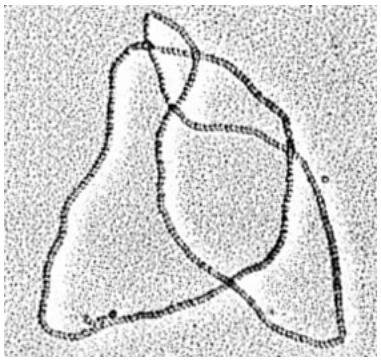
$K \subset R^3$ curva chiusa (a meno di deform. topologiche)

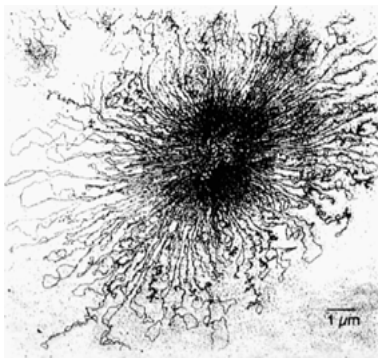












$c(K) =$ numero minimo di incroci dei diagrammi di K

Nodi geometrici

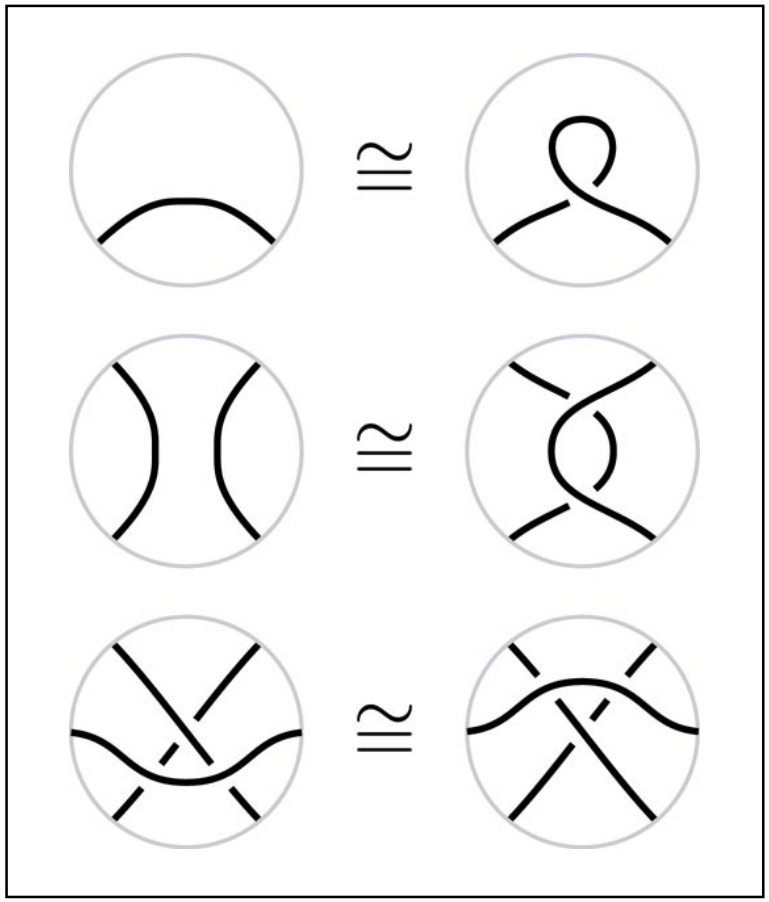
$K \subset R^3$ curva chiusa (a meno di isometrie euclidee)

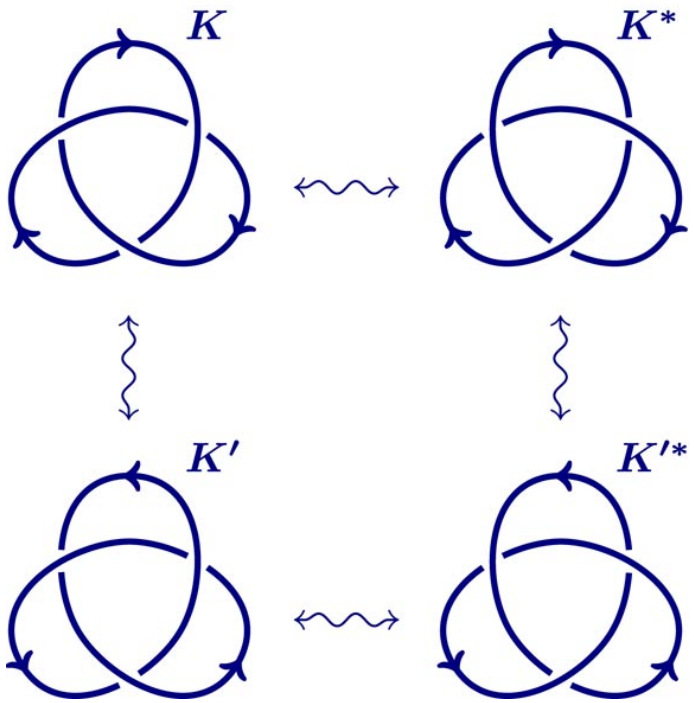
$$E = \int \left(\frac{1}{|\alpha(s) - \alpha(t)|^2} - \frac{1}{|s-t|^2} \right) dsdt$$

$C(K) =$ numero medio di incroci delle proiezioni di K

Isotopie e simmetrie

Movimenti di Reidemeister

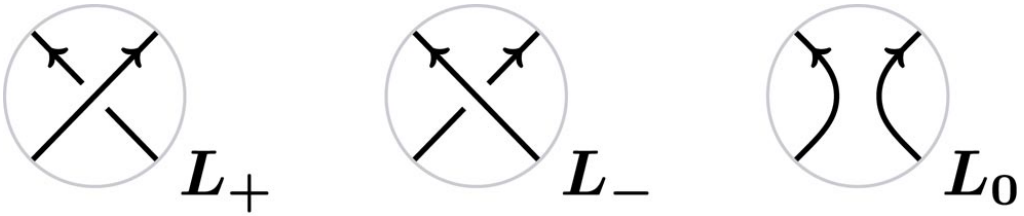




$K \cong K'$ (K invertibile)

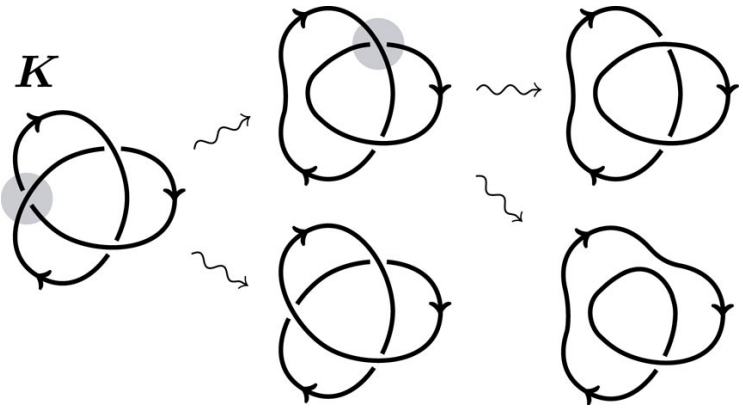
$K \ncong K^*$ (K chirale)

Polinomio di Jones



$$V(\text{nodo banale}) = 1$$

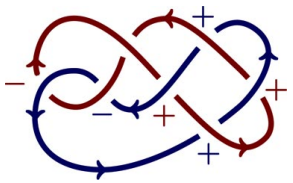
$$t^{-1}\,V(L_+) - t\,V(L_-) = (t^{-1/2} - t^{1/2})V(L_0)$$



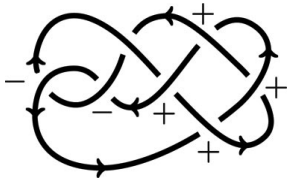
$$V(K) = -t^4 + t^3 + t$$

$$V(K^*) = -t^{-4} + t^{-3} + t^{-1}$$

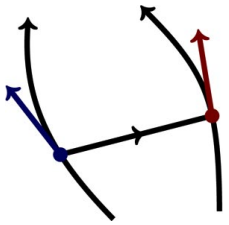
Indice di allacciamento



$$\mathbf{Lk}(K_1, K_2) = \left|\left\{\quad\right\}\right| - \left|\left\{\quad\right\}\right|$$



$$= \frac{1}{2}\left(\left|\left\{\quad\right\}\right| - \left|\left\{\quad\right\}\right|\right)$$

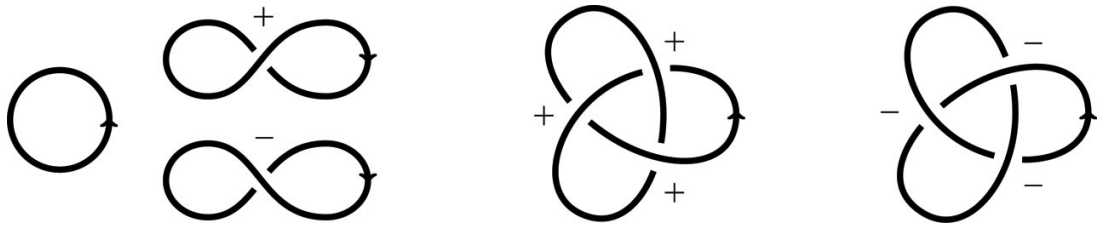


$$= \frac{1}{4\pi} \quad \frac{\alpha(s) - \beta(t)}{\|\alpha(s) - \beta(t)\|^3} \cdot \alpha'(s) \times \beta'(t)\, dsdt$$

$\mathbf{Lk}(K_1, K_2)$ è un invariante topologico

$$\mathbf{Lk}(K_1', K_2) = \mathbf{Lk}(K_1, K_2') = -\mathbf{Lk}(K_1, K_2)$$

Indice di contorcimento



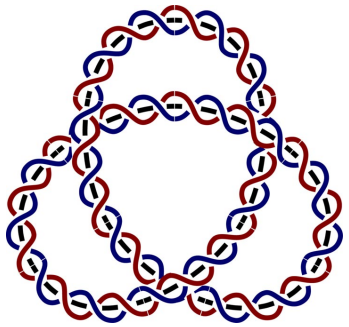
$$\mathbf{Wr}(K) = \frac{1}{4\pi}\int_{S^2}\left(\left|\left\{\quad\right\}\right| - \left|\left\{\quad\right\}\right|\right) d\sigma$$

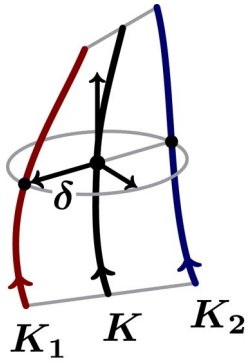
$$= \frac{1}{4\pi}\int \frac{\alpha(s)-\alpha(t)}{\|\alpha(s)-\alpha(t)\|^3}\cdot \alpha'(s)\times\alpha'(t)\, dsdt$$

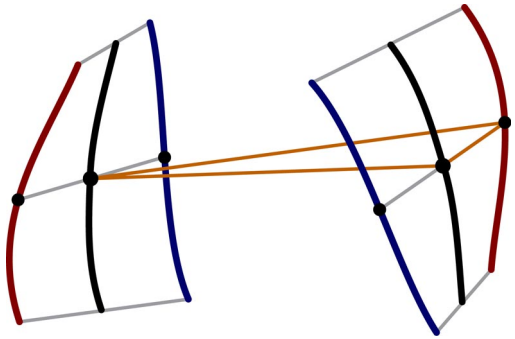
$\mathbf{Wr}(K)$ è un invariante geometrico ($|\mathrm{Wr(K)}| < C(K)$)

$\mathbf{Wr}(K') = \mathbf{Wr}(K)$ e $\mathbf{Wr}(K^*) = -\mathbf{Wr}(K)$

Il teorema di White







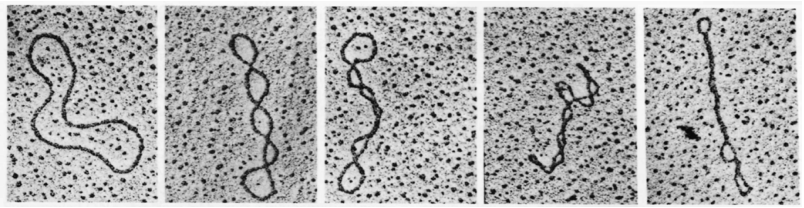
$$\mathbf{Lk}(K) = \mathbf{Lk}(K, K_{1,2}) = \mathbf{Lk}(K_1, K_2)$$

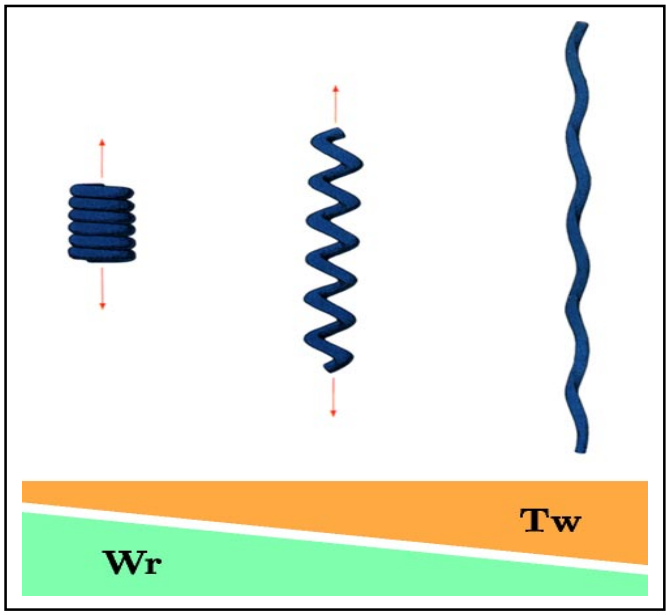
$$\mathbf{Tw}(K) = \frac{1}{2\pi} \int \frac{\delta'(s)}{\|\delta(s)\|^2} \cdot \delta(s) \times \alpha'(s)\, ds$$

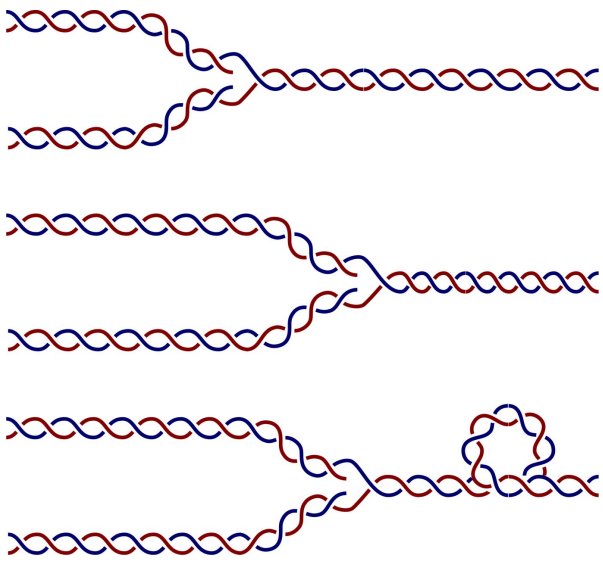
$$\boxed{\mathbf{Lk}(K) = \mathbf{Wr}(K) + \mathbf{Tw}(K)}$$

Super-avvitamento

$\mathrm{Wr}(K_0) = 0 \quad \mathrm{Tw}(K_0) = N/10.5$





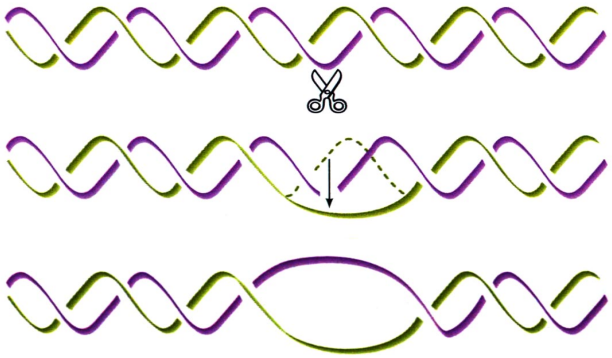


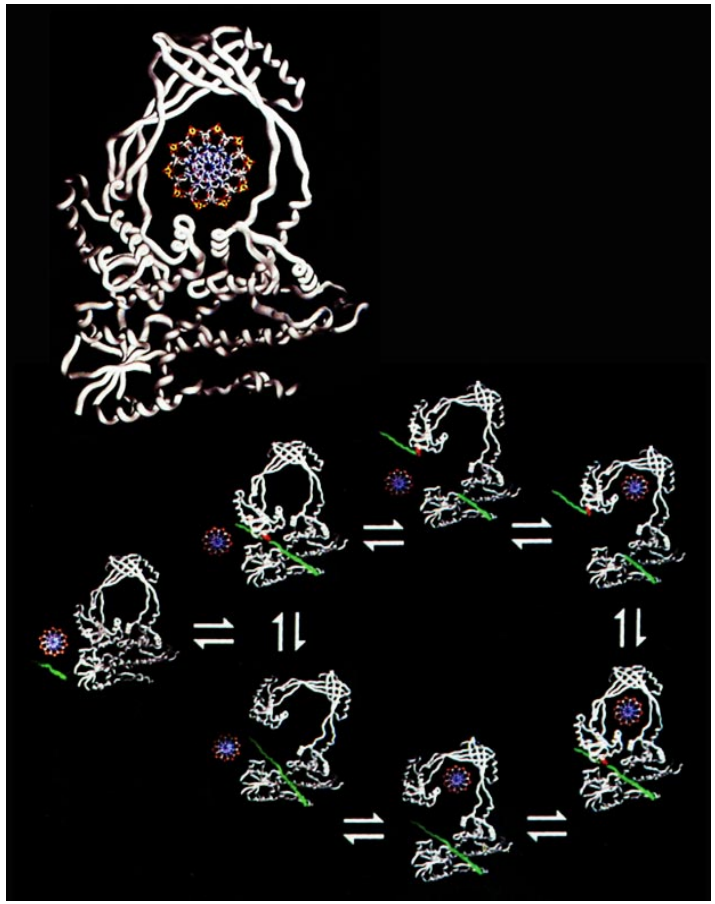
$$\sigma(K) = \frac{\mathrm{Lk}(K) - \mathrm{Lk}(K_0)}{\mathrm{Lk}(K_0)}$$

$\sigma(K) \sim -6\% \ (3/4 \ \mathrm{Wr} + 1/4 \ \mathrm{Tw})$

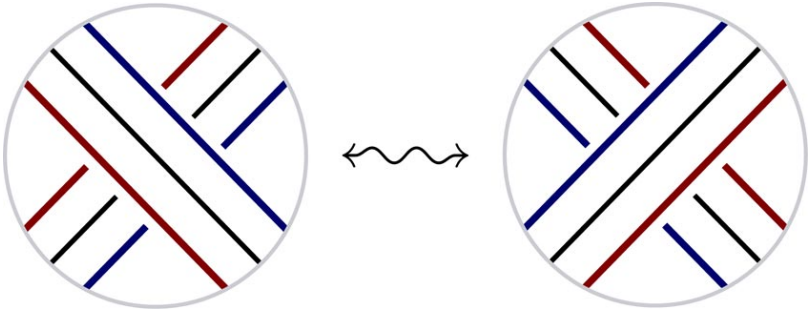
Topoisomerasi I

$$\Delta Lk = \pm 1$$
$$\Delta Wr \sim 0$$
$$\Delta Tw \sim \pm 1$$

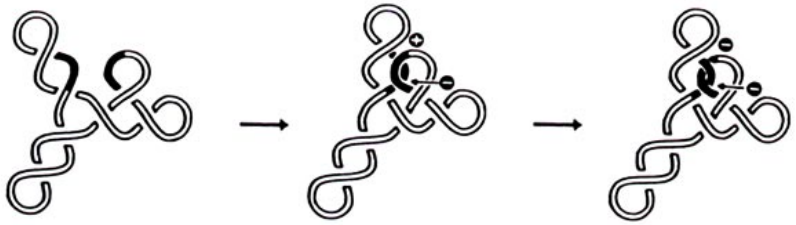


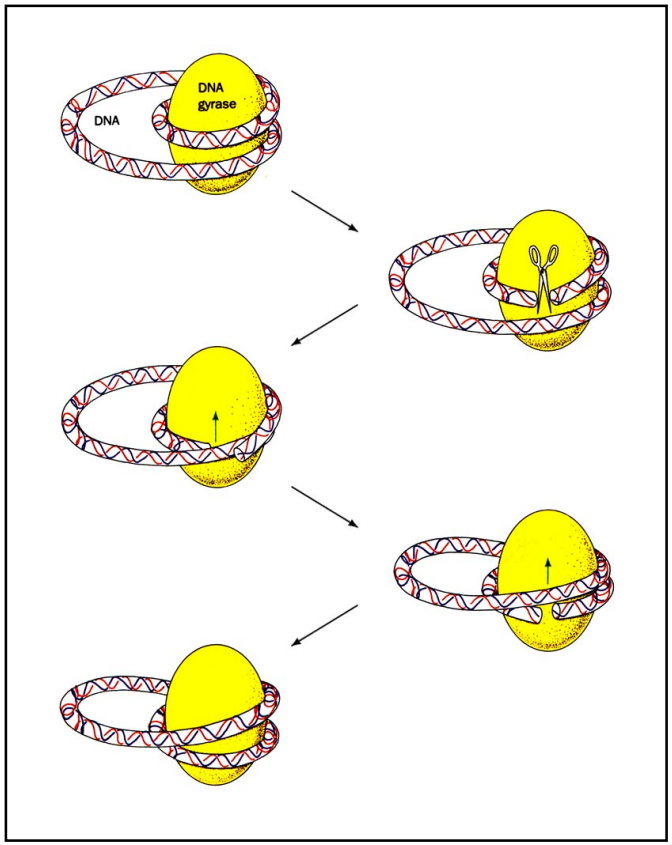


Topoisomerasi II

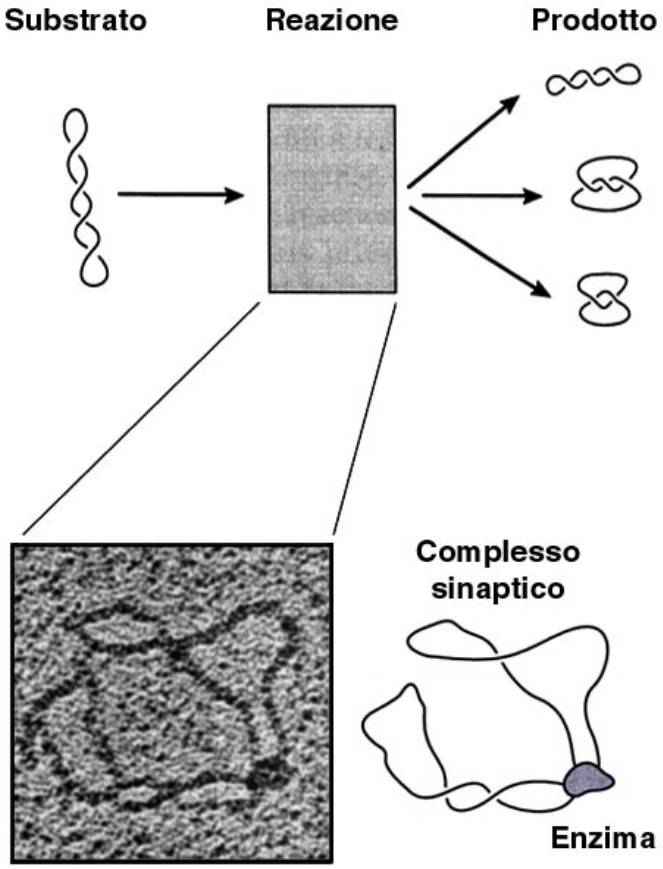


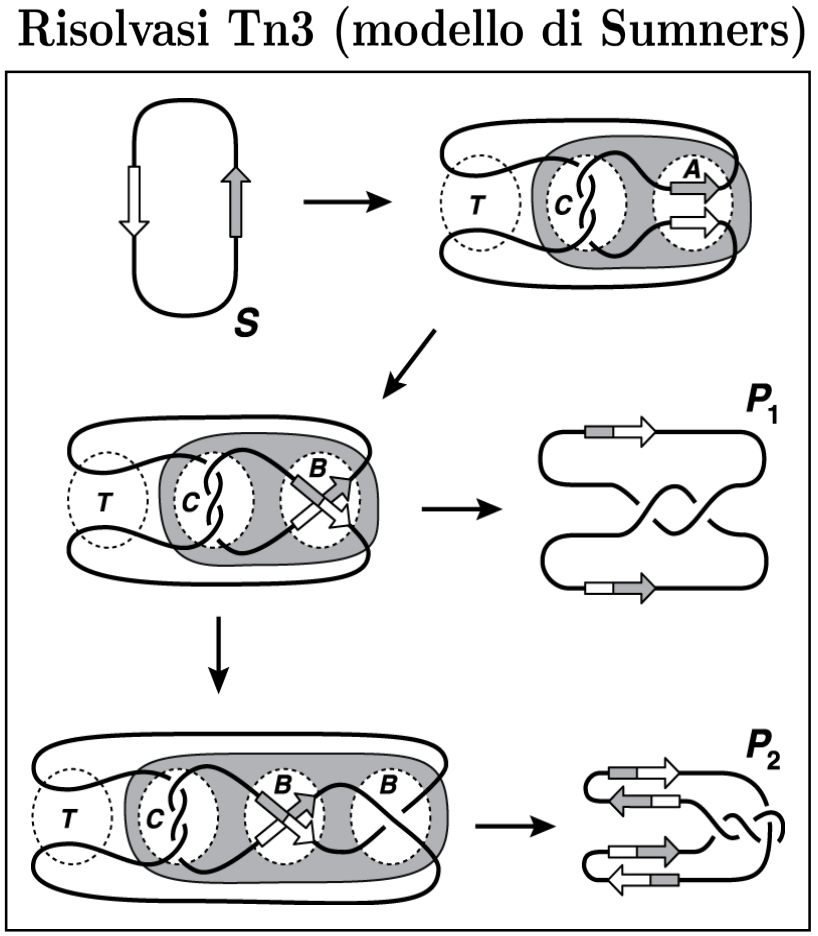
$$\Delta \mathrm{Lk} = \pm 2$$
$$\Delta \mathrm{Wr} \sim \pm 2$$
$$\Delta \mathrm{Tw} \sim 0$$



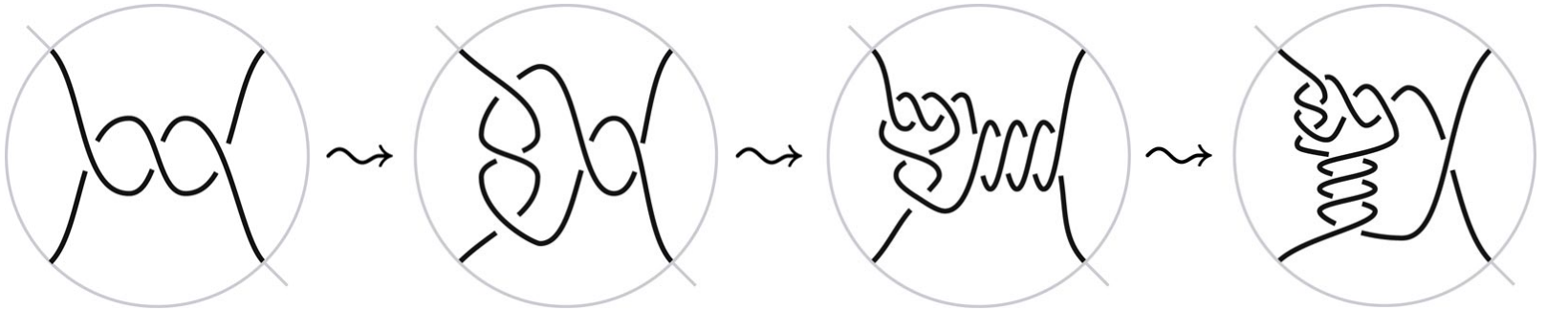


Enzimi e topologia

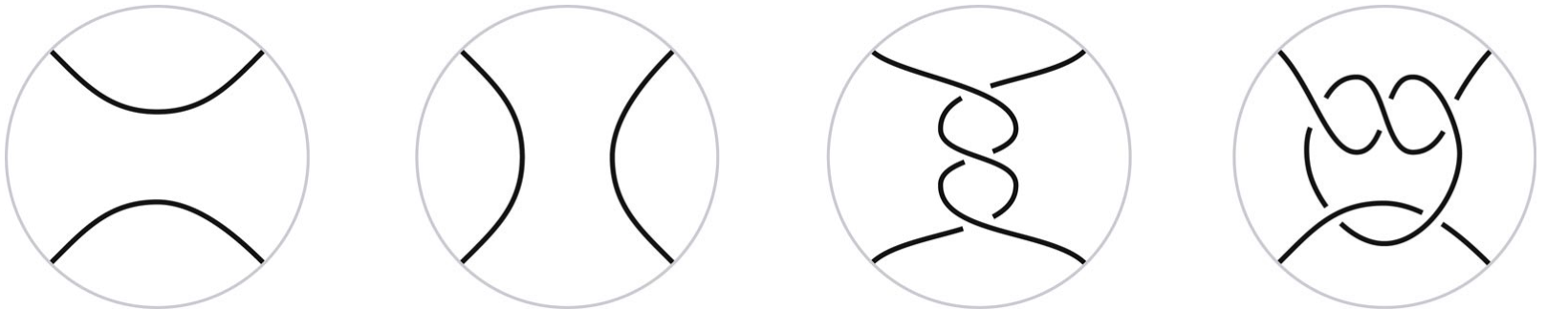




Matasse razionali

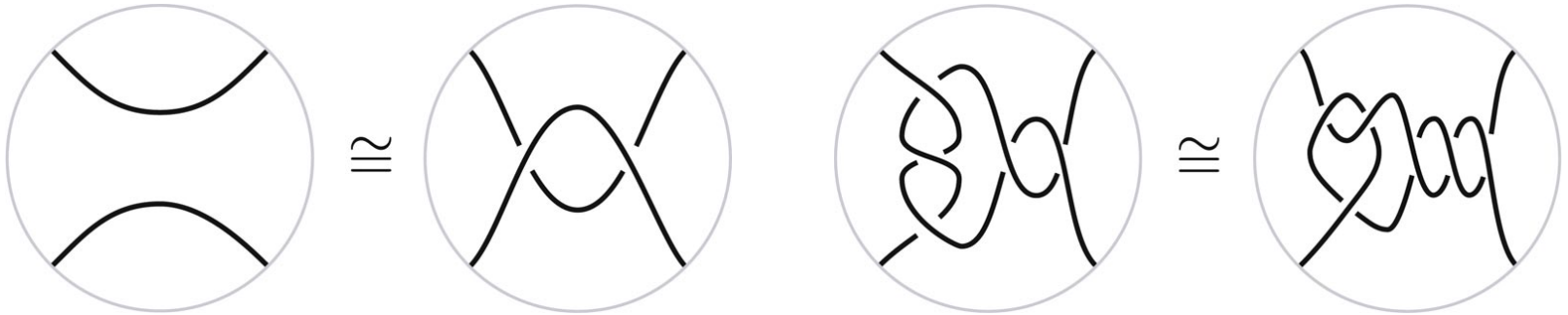


$T(3)$ $T(2,3)$ $T(-4,2,3)$ $T(-1,-4,2,3)$



$T(0)$ $T(0,0)$ $T(0,3)$ non razionale

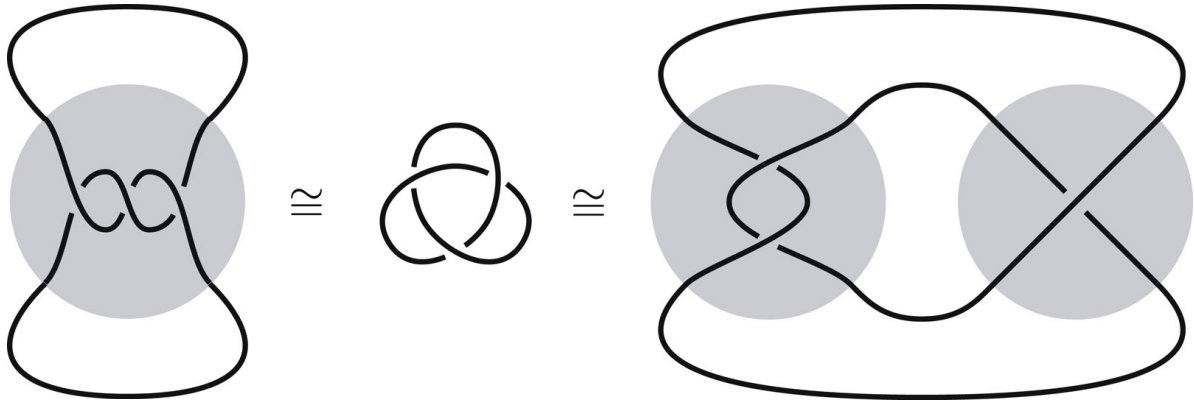
Il teorema di Conway



$T(0)$ $T(1,-1)$ $T(2,3)$ $T(3,-1,-2)$

$$T(a_1, a_2, \ldots, a_n) \leftrightsquigarrow a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \cfrac{1}{a_{n-1} + \cfrac{1}{a_n}}}}$$

Equazioni topologiche



$N(3)$ $N(-1/2 \dot{+} -1)$

$$N\left(\frac{a}{b}\right) \cong N\left(\frac{c}{d}\right) \Leftrightarrow a = c \text{ e } b = d^{\pm 1} \bmod a$$

$$N\left(\frac{a}{b} \dot{+} \frac{c}{d}\right) \cong N\left(\frac{ad+bc}{a'd+b'c}\right) \text{ con } a'b - ab' = 1$$

Matasse e DNA

Reazione: $nS \rightsquigarrow n_1P_1 + n_2P_2 + n_3P_3 + \dots$

Separazione per elettroforesi: $v_{P_k} \propto C(P_k)$

Determinazione di T **e** $P_1, P_2, \dots$

$$\begin{cases} N(C \dot{+} A) = S \\ N(C \dot{+} k\,B) = P_k \end{cases} \quad \rightsquigarrow \quad (A), B, C$$

Tn3 $(T = 0)$

$$\begin{cases} N(C \dot{+} A) = \\ N(C \dot{+} B) = \\ N(C \dot{+} 2B) = \\ N(C \dot{+} 3B) = \end{cases} \quad \rightsquigarrow \quad \begin{cases} A = \\ B = \\ C = \end{cases} \; ?$$